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ABSTRACT

Major revision in the U.S. mathematics curriculum since the 1960s have led to significant differences between the mathematics curriculum of the United States and those of many other countries. This study explored how eight Chinese immigrant students, with different cultural backgrounds, mathematics knowledge, and learning styles, learned in an Algebra I course. Three research questions were addressed: (1) How did students learn from the teachers and textbook? (2) What were the students' understandings of the algebraic concepts introduced in the class? (3) How did the mathematics they acquired in their home country affect their learning of the U.S. mathematics curriculum? A multiple-case study method was employed which permitted an examination of results across a number of cases (individual students) and generalization of findings. Data were collected by classroom observations, teacher/student interviews, testing, clinical studies of students' heuristic processes in problem-solving, and by other methods. Results are reported and discussed under the following categories: teachers (indicating how teachers can affect curriculum), text, student characteristics, the word association test, the sorting test, understanding versus rules, effect of pre-U.S. mathematics learning, problem-solving characteristics, and graph comprehension. Recommendations related to teacher education, mathematics texts, problem solving, and teaching immigrant students are included. (Author/JN)

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MATHEMATICS LEARNING STYLES OF CHINESE IMMIGRANT STUDENTS

FINAL RESEARCH REPORT

May 31, 1983

Principal Investigator:
Sau-Lim Tsang

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ABSTRACT

Major revisions in the U.S. mathematics curriculum since the 1960s, have led to significant differences between the mathematics curriculum of the U.S. and those of many other countries. This study explored how a group of Chinese immigrant students, with different cultural backgrounds, mathematics knowledge, and learning styles, learned in an Algebra I course. Three research questions are addressed: (1) How did the students learn from the teacher and textbook? (2) What were the students' understandings of the algebraic concepts introduced in the class? (3) How did the mathematics they acquired in their home country affect their learning of the U.S. mathematics curriculum? The research employed the multiple-case study method which enables the researcher to examine the results across a number of cases (individual students) and to generalize the findings.

The study was conducted at San Francisco's Mission High School. Data collection began in October of 1981 and ended in June 1982. The eight target students were selected from a Chinese/English bilingual Algebra I class. Data were collected by a variety of methods—classroom observations, teacher interviews, student interviews, testings, and clinical studies of individual student's Heuristic processes in problem solving. Additional data were also collected from school records, test results, and the student's classwork and homework.

The findings of the study are included in the final research report submitted to the National Institute of Education and in the paper "Mathematical Learning Styles of Chinese Immigrant Students: A Summary of Research Findings" presented at the 1983 Annual Conference of the American Association for the Advancement of Science.

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I. INTRODUCTION

The U.S. mathematics curriculum underwent major revisions in the 1960s (National Council of Teachers of Mathematics, 1969). First, the content was reorganized. Many topics which were previously taught in college are now introduced at the elementary or secondary level, and they are more integrated to convey the unified structure underlying mathematics concepts. Second, there was a shift in emphasis from mechanic operations to conceptual understanding. Last, there was increased attention to problem-solving skills. The changes were were dramatic, their effects lasting. Despite criticism in the late 1970s that this altered mathematics curriculum contributed to the decline in students computational skills, the reform have remained largely in place for the last fifteen years.

Many other countries have also revised their mathematics curricula. But the revisions were not always the same in scope or direction as those in the U.S. Today, there are significant differences between the mathematics curriculum in the U.S. and those of many other countries.

Immigrant students from such nations may experience difficulties in adjusting to the U.S. mathematics curriculum. They may have different learning styles, problem-solving techniques, and understandings of mathematics concepts which are attributable to the educational systems in their countries of origins. Of course, many of them may also have problems learning mathematics in the English language. To enable these students to "mainstream" into mathematics courses here, a special curriculum must be designed. Before one can be developed, however, research must be conducted on how immigrant students' mathematics knowledge and learning styles may be country-specific. This study attempted to explore the differences by examining a group of Chinese immigrant students studying Algebra I in the U.S.

CHINESE AMERICANS

1980 census figures show that there are more than 800,000 Chinese Americans in the United States. Between 1960 and 1980, the number of Chinese Americans grew phenomenally, increasing over 200 percent from 250,000 in 1960 to 800,000 in 1980, one of the largest proportional increases of any ethnic or racial group in the United States during that period. The major reason for this increase is heavy immigration and massive resettlement of refugees from Southeast Asia.

The history of Chinese Americans began in 1850, when the Chinese came to California during the Gold Rush. From 1850 to 1882, large numbers of Chinese laborers immigrated to the United States, usually to the West Coast and Hawaii, to seek a better life. They provided cheap labor which made possible the dynamic economic growth and development of the American West. The Chinese population in the U.S. grew until 1890, when census reports indicated more than 100,000 Chinese living the U.S. (Coolidge, 1969).

With the decline of the Gold Rush boom, the Chinese population became the scapegoat for economic problems because of their high racial visibility, cultural dissimilarity, and lack of political power. Agitation among trade unions, politicians and others resulted in housing, employment, and legal discrimination against Chinese in the U.S. and culminated in exclusion acts barring Chinese laborers from entering the U.S. from 1882 to 1943. Because of the Chinese Exclusion Act, the number of Chinese in the U.S. from 1882 to 1943 declined. However, the American policy towards Asian immigration shifted sharply in 1965 when the Immigration and Nationality Act was amended to reset annual quotas to 20,000 immigrants from each country up to 170,000 maximum from the eastern hemisphere (U.S. Commission on Civil Rights, 1980). After this policy change, Chinese immigration to the U.S. resumed.

In addition to immigrants, the U.S. also admits refugees into this country. Each year between 1955 and 1978, 40,000 refugees were permitted to resettle in the United States. With the expansion of the Indochinese program in 1979, the annual influx of refugees to this country increased tremendously. In 1980 alone, 156,000 refugees made their homes in the United States. The total number of Southeast Asian refugees in the United States was 555,546 as of October 1981, and this group is comprised largely of Sino-Vietnamese who fled their homeland to avoid political oppression. They will continue to come to the U.S. in the coming years if Congress permits, as conditions in the Indochinese peninsula remain in flux.

This study focused on a small group of eight Chinese immigrants students from Hong Kong, Mainland China, and Vietnam, the three countries from where large numbers of Chinese immigrants hail. The research method employed was the case study. The students were selected from a public high school in San Francisco. They were followed and studied for the entire 1981-82 school year. Large amounts of quantitative and qualitative data were collected on each student. The data were analyzed using each student as an entity and then examined across the eight students to arrive at generalized findings.

II. RESEARCH METHODS

OVERALL APPROACH

This study investigated the processes by which minority students learn in Algebra I. In particular, the research focused on a group of Chinese immigrant students from three different cultural backgrounds enrolled in an Algebra I class.

Three research questions were addressed by the study: (1) How do the students learn from the teacher and the textbook? (2) What are the students' understandings of the concepts introduced in Algebra I? (3) How does the mathematics knowledge acquired by the students in their mother countries affect their learning of mathematics in the U.S.?

To answer these questions and examine the complex interactions of the many factors (e.g. the students' education and cultural history, the political and social events which affected their educational aspirations, etc.) which affect the students' performance, this research employed the case study method. The object of inquiry of this research method — the case — is a certain bounded system of interest (Stake, 1978). In this study, each case is a student. This method does not entail the use of any particular type of data collection technique or an interest in any particular type of data. Instead, the study collected a large variety of qualitative and quantative data on each student by different means. Yin (1981) suggests that the case study is the preferred research method when "an empirical inquiry must examine a contemporary phenomenon in its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (p.98).

There are two types of case study designs: the single-case design and the multiple-case design. The latter was the one used by this study. The multiple-case design enables the combination of results from a number of case studies to generalize the findings. However, the multiple-case design should not be confused with experiments using small samples. The latter is interested in the means and averages of the whole group as the results, while the multiple-case design treats each case as a separate entity and seeks explanatory findings from the within-case evidence. The findings are then examined across cases to develop explanatory patterns for generalization (Kennedy, 1979; Green and David, 1981).

OVERVIEW OF THE STUDY

This study was conducted in San Francisco's Mission High School. The study began in October of 1981. Twelve Chinese students, who had recently arrived in the United States, were selected from the Algebra I class as target students for case studies. The class was a Chinese/English bilingual Algebra I class in which all students were classified as Limited English Proficient (LEP). The class was considered a college preparatory course and the students were placed in this class by school counselors who had examinedthe pupils' scores on a mathematics diagnostic test. However, during the school year, four students dropped out from the study for various reasons; and the study ended up with eight students.

Data were collected in three different periods (phases) of the school year by a variety of methods — classroom observations, teacher interviews, student interviews, testing, tape recordings, and observations of individual student's heuristic processes in problem-solving. Additional data were also collected from school records, test results and the student's classwork and homework. The following is a detailed description of the data collection procedure.

PROCEDURES

Target Student Selection (September 14 - September 31, 1981)

Twelve students from the Algebra I were selected according to the following procedures:

a. A letter requesting permission for the students to participate in the study was sent to the students' parents or guardians. The principal of Mission High School agreed to cooperate and issued the letter in his name. This underscored the legitimacy of the study and promoted the cooperation of the parents. The letter was written bilingually.

b. The Algebra I class was observed by the researcher for five school days. Students found to be problematic or unlikely to cooperate were eliminated. (A large amount of data were collected during the five days of observation. This data became part of the complete data set for the entire study.)

c. The Algebra I class was administered the ten-item short Arithmetic Reasoning Test. This test was adapted from the French Kit of Reference Tests for Cognitive Factors, Form R-4, Part I. It is designed to measure one's ability to determine what numerical operations are required to solve problems without actually having to carry out any computation. It has been shown to be a good measure of abstract reasoning and correlates highly with general IQ. The bilingual version of the test used was found to be satisfactory by Ng and Tsang (1980) and is reproduced in Appendix A. Those students who scored extremely poorly on this test were eliminated from the sample of this study.

d. Students were individually interviewed about their family backgrounds, the types of mathematics they had learned, and how they were taught in their native countries. Since in most countries other than the U.S., mathematical subjects (i.e., algebra, geometry, etc.) are not taught separately in different years, most students in this bilingual mathematics class have already been introduced to algebra.

e. Based on these results, 12 target students (6 male and 6 female) were selected to represent the variations in mathematics abilities, the different countries of origin, and both the refugee and immigration experiences of the total Chinese immigrant student population.

Phase One Data Collection (November 1 - December 16, 1981)

(a) Teacher Interview. The Algebra I teacher, Mr. Wong (a pseudonym), was interviewed and the following information was elicited:

- educational background,
- teaching experience,
- course objectives
- class organization,
- understanding of the course content,
- understanding of the students' difficulties and their special needs,
- pedagogical approaches, especially those adapted to meet the needs of the immigrant students,
- informal evaluation of the target students' progress,
- instructional objectives during the next phase of the study, and
- reason for changes in the original curriculum

The interview was informal but was guided by a list of questions. It took place over lunch at a restaurant near the school and was tape recorded for later transcription. During the interview, the researcher also briefed the teacher on his impression of the class and the target students, asbased on classroom observations and test data analyses.

(b) Classroom Observation of Target Students. The researcher visited the Algebra I class for five consecutive days and observed the target students. General ethnographic techniques were used during these observations. Data were collected in the form of field notes. Copies of the classwork completed by the target students were collected. The target students were observed on how they interacted with the teacher, with other students while doing classwork, and how they completed their class assignments. The researcher took particular note of how the target students responded to questions requiring memorization of rules, inductive thinking, and deductive thinking.

c) Individual Work Sessions with Target Students. The researcher collected data on the target students from individual work sessions. Each individual session lasted approximately one and one-half hours. These sessions were conducted after school at the convenience of the students.

During the sessions, students were interviewed, then administered a word association test and an algebra test. During interviews, the students were asked to recount the difficulties they encountered in the algebra text and in class. They were also asked to comment on the instructional methods of the teacher and their progress. The interviews lasted from fifteen to thirty minutes.

The word association test consisted of twenty-two pages (Appendix B). A different algebraic word/concept was printed at the top of each page. The students were asked to write down, within a minute, as many mathematical words/concepts as they could think of relating to the

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printed word/concept. The students were given two practice examples before the actual test. The instructions and the word association test took approximately thirty minutes.

The algebra test consisted of twelve items selected from the first three chapters of the algebra text (Appendix C). These three chapters were covered in class before the Thanksgiving holiday. Except for the first two items which were simple "warm-up" problems, the remainder of the items were selected because they had presented difficulties to the students in their homework and during the classroom observations. To solve these problems, the students had to apply the mathematics concepts introduced to them in the Algebra I class. The students were asked to use the think-aloud method to solve the problems. They were asked to solve the problems in front of the researcher, who was taking notes of the students' written work, thought processes, and other relevant data. The researcher also probed the students frequently and asked them to explain why they took certain approaches or how they arrived at a correct or incorrect solution. The algebra test took from one-half to one full hour. These individual sessions were tape recorded and were completed between November 30 to December 16, 1981.

d) Other Data Collection. Throughout the Phase One data collection, the Algebra I teacher cooperated by making available to the researcher the target students' classwork and homework. The researcher made copies of these materials for analysis.

Phase II Data Collection (March 1 - March 31, 1982)

(a) Teacher interview. The mathematics teacher, Mr. Wong, was interviewed for the second time. The researcher, by this time, was quite familiar with Mr. Wong's background and had established a good rapport with him. The interview was thus shorter than the first one. The researcher asked the teacher about the progress of the target students, about his feelings towards the difficulties encountered by the students, and any changes in the course content.

(b) Classroom observation of target students. Each target student was observed for one class period. Since two of the target students were transferred during the second semester into two other Algebra I classes taught by a Ms. Hall (pseudonymn), they were each observed for a period in those classrooms.

(c) Individual student work sessions. Individual work sessions were scheduled with the students either after school hours on campus or during the weekend at a public library. Each session lasted one to one and one-half hours. The students were interviewed on their mathematics experiences since the last work session, difficulties encountered, and areas that they wanted the researcher to help them with. The two students who transferred to Ms. Hall's Algebra I classes were especially asked to comment on their impressions of the different teaching styles of the two teachers.

After the interviews, the students were given an algebra test which consisted of 20 items. This test (Appendix D) was constructed in the same way as the one administered in Phase I with items reflecting the content taught during the past three months. The students were asked to solve the problems using the think aloud method.

Phase III Data Collection (June 1 to June 30, 1982)

(a) Teacher interview. Mr. Wong was interviewed on the last day of school. The procedure was similar to the Phase II interview.

(b) Classroom observation of target students. The target students in Mr. Wong's class were observed during the last two weeks of the semester. However, the classroom activities of those two weeks consisted of self-review with the teacher giving individual instruction to the students who requested help. With most of the class engaged in seat work, not much interaction was observed. Ms. Hall did not allow the researcher to observe her classes during the Phase III data collection.

(c) Testing. The target students were gathered in the afternoon on the last day of school and administered the Cognitive Ability Test and the Piagetian Picture Test. These were administered by request of two similar studies on minority students. The results of these tests were used mainly for comparison across the three studies. The Cognitive Test (Appendix E) consisted of 24 item on analogies, 26 items on computation, and 26 items on classification. The students were given 10 minutes for each of these sections. The Piagetian Picture Test (Appendix F) consisted of 72 item on Piagetian types of conservation and perception concepts. The test was not timed.

(d) Individual student work session. The researcher scheduled individual work sessions with each of the students during the four days after school ended. These sessions were conducted at a public library or at the students' homes and lasted from one to one and one-half hours. During the sessions, the students were administered the Sorting Test and Phase III Algebra Test. The Sorting Test consists of twenty-eight mathematics concepts, each written on a 3"x5" index card. The students were asked to sort the cards into groups. These concepts were those learned in the Algebra I class during the school year and are in Appendix G. The Phase III Algebra Test (Appendix H) consisted of fourteen items selected with the process described in Phase I. The items emphasized the content from the latter portion of the course. The students were asked to solve these problems using the think aloud method.

After the testing, the students were interviewed for their experiences during the last third of the Algebra I class and their overall impressions of the course. They were again asked to comment on the teachers and their teaching methods. The students were also asked to loan the researcher any notes or work samples they had accumulated from the Algebra I class during the school year. These were photocopied and returned to the students.

These three phases complete the data collection activitites of the study.

III. DATA ANALYSIS

As was discussed in the previous chapter, the research design used in the study was the multiple case study design. The general data analysis approach under this design was to treat the data collected from each student — case — separately and to analyze them independently. The focus was to seek explanatory findings for each student's learning difficulties. The findings were then examined across cases to detect patterns for generalization. The following is a description of how the study analyzed the variety of qualitative and quantitative data collected on each student.

INTERVIEW AND OBSERVATION DATA

These included the teacher interviews, the classroom observations, and the student interviews. These data were assembled. Field notes from observations and interviews were typed, indexed and stored on floppy diskettes. Similarly, tape recordings were transcribed, indexed and stored.

The researcher conducted preliminary analyses of the data as they were collected during each phase. The observation and interview data were then sorted according to different combinations of the indices. The researcher examined these data with the target students' classwork and homework. Salient aspects of the students' learning styles and difficulties were noted and used in the preparation of the next phase's interview questions, problems and observation schemes.

At the end of the Phases III data collection, these data were re-examined for overall explanatory evidence.

WORD ASSOCIATION DATA

The word association test was designed to examine the mathematical cognitive structure, that is, the relationships among the mathematical concepts in the students' memories. The data were analyzed by calculating the relatedness coefficient matrices and by using the multidimensional scaling procedure. The relatedness coefficient (RC) incorporates the response frequency to a given stimulus word and the overlap between response distributions for pairs of stimulus words. These coefficients, ranging from 0 to 1 inclusive, indicate the degree to which two concepts are related in the student's memory. The larger the value of the coefficient, the closer the relationship between the two concepts. The formula for calculating the RC is :

$$RC = \frac{\overline{A} \cdot \overline{B}}{(A \cdot B) - [n^P - (n-1)^P]^2}$$

where A is a row vector giving the rank of the n words in A (the longer list); B is a column vector giving the rankings of words in the shorter list where the first term is again ranked as "n";

$\overline{A}$ is a row vector whose elements are the rank order of words under A shared in common with B; $\overline{B}$ is a column vector giving the rank order of these words in B;

P represents some fixed number greater than zero which may be determined from the shape of the probability distribution of the responses. P was set equal to 1 in this study; all portions of the subjects' response distribution received equal weight.

The word association data on each subject were converted to a 20x20 matrix by obtaining the relatedness coefficients between each pair of the 20 algebraic concepts. The matrix is called the relatedness coefficient matrix (RC matrix). These RC matrices are symmetric.

When scaled, the RC matrices represent points such that the interpoint distances correspond in some sense to the relatedness between the algebraic concepts. Two-dimensional scalings of the RC matrices were obtained and the scaling solutions indicate graphically how these concepts were clustered in the student's memories.

PROBLEM-SOLVING DATA

The problem solving data were collected from the students who used the think aloud method to solve the three algebra tests. The data came in the forms of notes taken by the researcher who observed the problem solving session, tape recorded the students' speech, and collected the worksheets the students used to solve the problems. These data were coded and the general heuristic processes used by each student were constructed. These processes were compared with the ones taught in the Algebra I class and the ones suggested by other researchers such as Polya (197) and Schoenfeld (1980).

SORTING TEST

No special procedure was used to analyze the Sorting Test data. They were recorded as soon as each student completed the test. The results of this test were compared to the results obtained from the Word Association Test.

OTHER TESTS

The other tests were analyzed by checking the students' answers with the keys to obtain the number of correct answers. The incorrect answers were also tabulated for error analysis.

IV. SETTING OF THE STUDY

The study was conducted at the Mission High School of the San Francisco Unified School District.

THE CITY

San Francisco has one of the largest populations of ethnic Chinese in the U.S. Since it is the port of entry for most Chinese immigrants, San Francisco has experienced all phases and waves of Chinese immigration. Many have settled permanently in San Francisco and the surrounding areas.

Since the passage of the 1965 Immigration and Nationality Act, Chinese immigrants have arrived at a steady rate from Hong Kong and Taiwan. Under this Act, they were permitted to enter the U.S. under the provisions for family reunification or skilled, professional workers. More recently, with the normalization of diplomatic relations between the United States and the Peking government, Chinese immigrants are coming from mainland China, usually under the family reunification category. And in recent years, the political turmoil in Vietnam has caused many Sino-Vietnamese to flee their homeland and settle as refugees to the U.S. These different waves of Chinese immigrants are all represented in San Francisco, where forty-five percent of the city's population is either foreign-born or children of foreign-born parents.

It is estimated that 38 percent of the city's population have less than a high school education, 45 percent have graduated from high school, and 17 percent are college graduates. Of employed workers over the age of 16, 25 percent are in professional or managerial capacities, 55 percent are in skilled occupations, and 20 percent are in semi and unskilled areas. A 1976 California Employment Development Department survey revealed that 15,116 families in this city were below the poverty level and that 44,113 persons in the labor force were unemployed.

THE SCHOOL DISTRICT

San Francisco has a unified school district with, at present, 15 high schools, 16 middle schools, and 76 elementary schools. Because of severe fiscal cutbacks, the district has had to terminate the employment of over 1000 teachers since the 1978-79 school year. Additionally, the afterschool recreational program and the summer school program have been severely curtailed.

The student enrollment in the San Francisco Unified School District (SFUSD) for the 1981-82 school year averaged 58,931 pupils. The ethnic makeup of SFUSD was as follows:

o	Spanish Surname	9,883	(16.8%)
o	Other white	10,255	(17.4%)
o	Black	14,017	(23.8%)
o	Chinese	11,358	(19.3%)
o	Japanese	718	(1.2%)
o	Korean	565	(1.0%)
o	American Indian	331	(0.6%)
o	Filipino	5,012	(8.5%)
o	Other non-white	6,792	(11.5%)

THE SCHOOL

Mission High School is located in an area of the city where the population is mostly Latino. However, the SFUSD is under court order to desegregate; each ethnic neighborhood is divided into sections and students living in different sections of a neighborhood are assigned to different schools around the city. Thus the student population at Mission High School is multi-ethnic. As of October 24, 1981, the student population of Mission High School was 1,190. Its ethnic breakdown was as follows:

o	Spanish Surname	682	(35.7%)
o	Other white	103	(5.4%)
o	Black	330	(17.3%)
o	Chinese	322	(16.9%)
o	Korean	1	(0.1%)
o	American Indian	12	(0.6%)
o	Filipino	244	(12.8%)
o	Other non-white	216	(11.3%)

Mission is also one of the high schools in San Francisco designated as a "Newcomer Center," where immigrant students are assigned when they first arrive. Special programs and staff are available in the Newcomer Centers to serve these students' language and other needs.

THE ALGEBRA I CLASS

The Algebra I class met during the second school period from 9:25 to 10:25. However, the last ten minutes of the class were reserved for school announcements from the public address system. The class was on the second floor of the school building. When school began, there were 31 students in the class and they were seated theatre style, in five columns facing the chalkboard. The male students were all seated on the left three columns and the females (except for one) on the right two columns of the room. The teacher's desk was located on one side of the front facing the class. Three other male students joined the class in mid-November. They took up three empty seats, one on the male side and two on the female side. The teacher, Mr. Wong, stated that the male and female separation was initiated by the students. Except for the three late-comers, he did not assign seats to the students.

Throughout the study, the class was always taught as a group. Mr. Wong usually began the class by lecturing. He would then ask some students to do some problems from the text on the chalkboard and he would ask other students to correct any errors. Other times, he would ask the students to indicate which problems were most difficult on the homework assignment from the previous day. Then he would ask some students to do these problems on the board or if time was short, he might do these on the board himself. Before the end of the class, he always assigned homework for the day. Students were usually very attentive during this Algebra I class, being silent when the teacher was lecturing. When some students were doing problems on the board, others tended to talk among themselves, but conversation usually involved the problems on the board or other homework problems. The noise level was never excessive. The teacher said that this class was exceptionally well behaved and that the early time of instruction helped to enhance the students' attentiveness.

The text used in the class was Algebra: Its Elements and Structure, Book I, Second Edition, by J.H. Banks, M.A. Sobel, and W.E. Walsh.

THE ALGEBRA I TEACHER

The teacher of the Algebra I class was Mr. Wong. He is an immigrant and has lived in the U.S. for almost twenty years, arriving in 1962 from Hong Kong to attend college. He studied at Fresno State University for two years and then transferred to University of Washington in Seattle where he graduated with a Bachelor of Science degree in physics. Then he spent a year and a half at the University of

Washington to complete enough credits to enter the graduate program in physical oceanography. Unfortunately, the funding for that program was decreased and the research division closed.

Mr. Wong then moved to the New York area. For a summer, Mr. Wong worked on a hospital research project which utilized knowledge of chemistry and physics. He then moved on to a teaching assistant position at Essex County College where he conducted a physics lab course for two years. There, he says, he was active in community affairs dealing with the school district and Chinese immigrant students.

In 1972, Mr. Wong moved back to the West Coast. Initially he taught Cantonese at Stanford University for one quarter. He noticed there was a demand for bilingual teachers so he began work as a bilingual teacher's aide at one of the high schools in San Francisco while he studied for his teaching credential at the University of San Francisco. After obtaining his teaching credential, he taught for one year at the district's Benjamin Franklin Middle School, which is located in a lower-middle class black neighborhood. Then he transferred to teach at Mission High School.

At Mission High, Mr. Wong has been teaching the college preparatory bilingual Algebra I class for 4 years. Because it is a college prep course, he felt it was necessary to stress basic concepts such as the real number line and factorization. He felt students must obtain a grade of C or better if they were to proceed successfully along the college prep track.

Mr. Wong's attitude was that math should be an analytical and not a mechanical process. He favored the inductive method. Once he had shown the students how to solve the problem, he expected the students to be able to do the work by analyzing the problem and asking questions. He wanted the students to be able to discover for themselves, where and why they made an error. Thus he guided them to the solution by continuously asking them questions.

Because the English comprehension levels of his students was limited, Mr. Wong found that he had to convey the text material largely through lectures in Chinese. However, his experience indicated that once the students had mastered the first five chapters, it was easier for them to understand the later chapters. The main problem he saw for the students was understanding the concepts and then being able to apply them to a problem. Examples of problem topics that he cited were absolute value, inequality, proofs, and theorem.

THE TARGET STUDENTS

Twelve target students were identified by the sample selection procedure and agreed to participate. However, during the Phase I data collection, three students dropped out claiming lack of time or other commitments leaving nine students. In the second semester, another target student left the school to attend a community college. The other eight students continued through the study and completed all three phases of data collection. The profiles of these eight students are included in their case studies.

V. FINDINGS

In this chapter, the eight case studies are synthesized and the results summarized. As was described in Chapter II, the multiple case study design of this research enabled the researcher to examine many factors related to the mathematical learning of the Chinese immigrant students. However, two caveats are required for the interpretation of the findings. First, the nature of this design (the small sample size vs. the large number of factors) limited the generalizability of the findings. Consequently, many of them should be considered as hypotheses for further validation.

Second, while the findings were generated from eight Chinese immigrant students, they are probably not unique to this group of students. The researcher provided an extensive description of the research setting and the target students to allow others to interpret the findings accurately and to apply to other settings and contexts.

THE TEACHERS

The study originally intended to study only one teacher, Mr. Wong. However, during the second semester, two of the target students were transferred into another Algebra I class taught by Ms. Hall. Ms. Hall was unreceptive to the study and at first, refused to let the researcher to observe her class. It was only after repeated requests that she reluctantly allowed the researcher observe her class twice. And of course, no interview was granted. Therefore, the data collected on Ms. Hall was limited and was largely based on the interviews of the two target students in her class.

The two teachers showed extreme differences in their teaching styles and their approach to the course content. Mr. Wong was completely in tune with the spirit of "modern" mathematics. He closely followed the content and approach of the textbook, placing a great emphasis on understanding. In his interviews, he said that it was more important for the students to understand the concepts than to simply know how to do the problems. He also said that the first three chapters of the Algebra I textbook were the most important part of the course for they introduced the fundamental algebraic concepts. The students had to master these three chapters. Otherwise they would fall further and further behind as the course progressed, and would have trouble taking more advanced courses. Mr. Wong liked to use the inductive method to teach. He said that the inductive method seemed to capture the students' attention and motivate the students to think more. The students characterized his instruction as clear but said that he alway made things profound and complicated. Mr. Wong was quite relaxed in the discipline of the class, and there was much student-to-student interaction, especially during seatwork. However, the class was usually very attentive during his lectures. He said that this was an exceptional class in behavior and not typical of other classes he had.

Ms. Hall, on the other hand, seemed to be the antithesis of the "modern" mathematics teacher. Students had said that she was not too good in mathematics and that she never taught any of the more advanced course. However, the researcher could not verified this information without Ms. Hall's cooperation. Though she used the standard algebra textbook, she freely omitted topics or chapters she considered unimportant. Usually, the topics she omitted were the fundamental algebraic concepts or those related to them. Thus, she only briefly touched on the first three chapters of the text, did not introduce the number line, and omitted all the materials on graphs. Her main objective for the course was to get the students to proficiently do problems involving factoring. She was very strict in discipline, and her class was always quiet during her lectures and seatwork assignments. Ms. Hall also emphasized that the students should follow the exact procedures she taught in the class and was concerned with the neatness of her students' work.

The extreme differences of these two teachers indicate the extent to which teachers influence the mathematics curriculum. Despite all the effort spent on curriculum revision in the last twenty years to emphasize understanding and teacher training, there are still classroom teachers teaching the algebra curriculum of the fifties.

THE TEXT

The textbook used in the Algebra I class was Algebra: Its Elements and Structure, Book I, Second Edition, by J.H. Banks, M.A. Sobel, and W.E. Walsh. The book is typical of many commercially available algebra texts. It begins by introducing the basic concepts of sets, the real number system, and the number line. Next, it utilizes the number line to introduce the four operations and their properties. Then it moves into open sentences, graphing linear functions, equations and inequalities, polynomials and factoring, quadratic equations and inequalities. The last two chapters of the book are on logic and trigonometry. Neither Mr. Wong nor Ms. Hall covered these chapters. Although the study did not specifically examine the textbook, it nevertheless came up with several observations that might guide future curriculum development efforts.

First, there are a large number of problems covering almost all aspects of the content. However, the number of problems related to each topic seemed to be arbitrarily determined and not based on empirical data on the students' need for practice. Maybe the authors had counted on the teachers to assess students' needs and to adjust the homework and classwork assignments accordingly. However, the two teachers observed in this study did not tailor assignments to the students' needs. Instead they usually assigned the odd numbered or the even numbered problems to the students, thus preserving the proportion of problems assigned to each topic by the textbook authors.

Second, several of the students complained about the concentration of the fundamental algebraic concepts in the first three chapters of the book. The range of new concepts in these initial chapters seemed to be overwhelming to the slower students; many of them were frustrated. After the first three chapters, these concepts gradually disappeared from the text and were correspondingly forgotten by students as the school year progressed. If the fundamental concepts were distributed across more chapters and integrated into other topics, instruction may be more effective.

Despite the treatment by the curriculum of the real numbers as a system with specially defined operations, the students did not seem to acquire this way of thinking. The four basic operations and their accompanying properties were still something associated uniquely and naturally with the real numbers and not an arbitrary system. This can demonstrated by the target students' performance on the following problems.

The operation * is defined as : a * b = 2a + b
Is * commutative? Why?
Is * associative? Why?

None of the target students could do this problem. In fact none of them could understand what * was. They were all perplexed and kept asking whether * was addition or multiplication.

Third, the chapter on graphing linear functions was ineffective. It introduced formulae on how to calculate slopes, distances, midpoint, etc. However, most students could not recall these formulae or confused one with another after a month and were unable to do the related problems. They were not able to derive the formulae or to check whether the formulae were the correct ones. Perhaps this was due to the emphasis of the chapter and its accompanying exercises on the application but not on the understanding of the formula.

THE STUDENTS

The characteristics of the students merit some discussion here. The Algebra I class was a Chinese/English bilingual class, and all the students were limited-English-proficient Chinese students. The course was the first in the college prep mathematics series offered by the school. However, observations and test results indicated the Chinese students in the class were quite varied in their mathematics abilities. The eight target students reflected this diverse mathematics background. The researcher interviewed these students on why they were taking the Algebra I class. Of the eight, only two indicated that their career choices required them to enroll in the college prep series. One student said that he excelled in literature in China and had wanted to be a writer, and the others did not indicate any preferences. They said that they were assigned to the Algebra I class by the school counsellor (a Chinese American) and were told that it was important to learn more mathematics since they were not proficient in English and all the career opportunities available to them, because of their limited English proficiency, required mathematics trainings.

The students were also characterized by their diligence and eagerness to learn. All eight students said that they usually spent long hours after school studying and doing the homework assignments. They also said that they actually read the examples in the Algebra textbook at home. They admitted that it was a difficult task and they would constantly use the dictionary to check the words that they did not understand. One student said that she read the entire chapter, text and examples, to ensure that she did not miss anything. This diligence probably reflects the immigrants' belief that they could gain upward mobility in the U.S. through education.

THE WORD ASSOCIATION TEST

The students' responses to the word association test were perplexing. The test asked the students to write down, within one minute, as many mathematical words/concepts as they could think of that were related to the stimulus words/concepts. Each student was given detailed explanations on the test and all of them responded correctly to the two practice items before the actual test. However, in the actual testing, five of the eight students failed to give the appropriate type of response to the test. Instead of writing down related items, they wrote down either explanations or examples of the stimulus words/concepts. Only three students answered appropriately.

One could attribute this behavior to the immigrant students' lack of familiarity with the unusual task demands. Or maybe their responses indicated that the students did not perceive mathematics concepts to exist in an interrelated structure. For them, each concept was independent.

However, three of the target students did give appropriate responses. This result was apparently not related to the students' mathematics ability. Of these three students, one was a high achiever from Hong Kong, one an average achiever from Saigon, and one a low achiever from Hong Kong. Classroom observations showed that the three did share one commonality: they were more eager to learn what was taught in the Algebra I class than to apply what was learned before they came to the U.S. They were also the ones who used more of the methods and procedures taught in the Algebra I course to solve the problems during the individual work sessions.

This result also indicated that being able to conceive mathematical concepts in an interrelated structure is not related to the students' understanding of the concepts. For example, two of the target students were among the best mathematics achievers but failed to give any appropriate responses to the Word Association Test, and one student who gave the most appropriate responses to the Word Association Test did not demonstrate a firm grasp of the algebraic concepts during the problem solving session. The students' inability to see the connection among various algebraic concepts might also be due to the exercises given in the text which were usually narrowly focused and required the application of only one concept.

THE SORTING TEST

This test was straightforward. It asked the students to sort twenty-eight words/concepts, each written on a 3 x 5 card, into related piles. The students had no difficulty in completing the task. The most interesting result was that the three students who gave the appropriate responses to the Word Association Test grouped the twenty-eight concepts into only six categories, while the others had from eight to ten groups. This implied that the twenty-eight concepts appeared to be more interrelated for these three students than for the others. The results of the sorting test correspond well with the Word Association results. One must note, however, that the Word Association Test was administered in December, the third month of the school year, when the students were first introduced to the algebraic concepts, and the Sorting Test was administered in June during the last week of school when they had two semesters of instruction and experience with the concepts.

UNDERSTANDING VS. RULES

While Mr. Wong focused his instruction on understanding concepts and did not teach rules which could expedite problem-solving, the high achievers, nevertheless, were able to discover many of these rules by themselves. They did so while undertaking the numerous homework assignments. But this discovery process did not hold true for the lower achievers who usually learned only what they were taught and were not able to derive the shortcuts by themselves. The following problem illustrates this:

Find the solution set $-7x < 4$.

The procedure taught in the class, which was also the procedure used by the lower achievers, was to find the solution to the equation $-7x = 4$ which yielded $x = -4/7$. Plot the point, $-4/7$, on the number line. Find any two points on both side of $-4/7$, substitute them back in the inequality $-7x < 4$ to determine the solution set. However, the two higher achievers did the problem by moving the coefficient -7 onto the right hand side of the inequality as the denominator. Because the coefficent was a negative value, they reversed the inequality sign. When interviewed, they explained that they had discovered the rule themselves; and they were able to show the researcher how they arrived at the rule.

Conversely, if the lower achievers had been taught only rules, they would probably learn to do the problems accordingly but without understanding the reasoning behind the process.

How about the higher achievers if they were taught only rules? They are likely to be able to discover the reasoning behind the rules and especially if the exercises facilitate the discovery. But due to the lack of such problems and tasks in the textbook, the discovery would not come as easily.

EFFECT OF PRE-U.S. MATH LEARNING

A common characteristic of the mathematics curriculum taught to the study's eight target students before they came to the U.S. was an emphasis on tests. The educational systems of these students' home countries were all very competitive. Every student had to pass a series of public examinations in order to gain entrance to higher levels of schooling. Thus, the main objective of the teachers was to help the students obtain good scores on the examinations. Mathematics was taught with the same objective; teachers concentrated on the students' ability to do problems quickly and accurately. The students, therefore, learned many rules and formulae to solve different types of problems expediently. Because they were accustomed to this type of orientation, the students were perplexed by the curriculum taught by the Algebra I teacher. In general, they said that the explanations were clearer, but the way they were presented made the concepts too profound and complicated. When the homework assignments could be done with the rules and formulae they had learned before immigrating, they would ignore the methods and procedures taught in the class and proceed with their own methods. Consequently, they would forget most of what was taught in the class.

Since the Algebra I teacher, Mr. Wong, was quite relaxed in his style and allowed almost free student-to-student interaction within the class, there was much peer teaching in the class. Consequently, many of the rules and formulae, though not taught by the teacher, were transferred from one student to another. Having acquired these rules and formulae to facilitate their problem solving, the students would abandon the procedures learned in class, which were usually more cumbersome.

A second aspect of their past experience in mathematics was student unfamiliarity with problems that did not ask for a specific solution or a set of solutions, that is, problems that asked for proofs or explanations. The students had seldom encountered such problems in their home countries, and many were lost when faced with such tasks. They did not know know to write proofs nor did they know how to write an explanation. For example, none of the target students was able to do the following problem.

Problem II-18: Is the following proof correct? Why?

$$\begin{gathered}
\text{Let } x = 2 \text{ and } y = 1. \text{ Then,} \\
x^2 - 2xy + y^2 = x^2 - 2xy + y^2 \\
x^2 - 2xy + y^2 = y^2 - 2xy + x^2 \\
(x - y)^2 = (y - x)^2 \\
(x - y) = (y - x) \\
2x = 2y \\
x = y \\
2 = 1
\end{gathered}$$

The following is a comparatively easy problem and most of the students gave the correct answer, none of them were able to write the explanation.

Problem I-3: True or false? Why?

$$\begin{gathered}
\text{If } a > 0,\ b < 0 \text{ and } |a| > |b|, \\
\text{Then } a + b = -(|a| - |b|)
\end{gathered}$$

Third, unlike the U.S., the students' home countries taught through algebra throughout the secondary school years as part of an integrated mathematics curriculum. Therefore, most of them had some dursory introduction to many algebraic concepts. This seemed to give a false confidence to the students, especially the higher mathematics achievers, who tended to think they had already mastered the subject and did not pay attention to the class.

PROBLEM-SOLVING CHARACTERISTICS

The eight target students broadly followed the general problem solving procedure suggested by G. Polya in his book, How to Solve It. The procedure consists of four steps: (1) try to understand the problem; (2) find the connection between the data and the unknown to develop a solution plan; (3) carry out the plan; and (4) examine the solution obtained. However, the students also used strategies worth mentioning.

Since all the eight students were of limited-English-proficiency, they would first read the word problems as they were written in English and then usually reread the word problems again in Chinese to ensure that they understood the problems. The ways to which their English proficiency affected understanding of the problems differed according to their mathematics abilities. For the higher achievers, English did not seem to affect comprehension much. Somehow, they were able to decode the mathematics underneath all the words. The lower achievers were more dependent on their English proficiency. They usually said that they did not understand the problem because there was one or more English words that they did not know. But often, after the researcher had translated every word to them, they still would not understand the problem.

Secondly, as was discussed earlier, the students tended to apply more rules and formulae than strategies such as trial and error or diagramming. But since there was no comparison group, this finding might be a function of the problems and not the students.

Lastly, except for one, none of the students had the habit of checking the results or solution. Maybe this was the result of training in their home country where speed was emphasized so that they could do as many problems as possible within a time limit, a skill that was essential to obtaining good scores in examinations.

GRAPHS

The topic of graphs was especially difficult for the students. The eight target students were not able to do most of the graph problems except for the straightforward ones which asked them to plot lines of equations. For this type of problem, they could follow the procedures of writing down several number pairs and drawing a line through the points. Any graph problems that were somewhat unusual would present many difficulties to the students. For example, none of the students were able to do the following problem:

> Problem II-5: Graph all points (x, y) such that x and y are both between 1 and 2.

One had the feeling after observing these students that they really did not grasp the relationship between two dimensional space and its mathematical representations. However, this difficulty in graphing might be true for all students and not only for these immigrant students.

RECOMMENDATIONS

Teacher education. The differences in the teaching styles and objectives of the two teachers observed in this study have profound implications for pre-service and in-service of mathematics teachers. If the "modern math" curriculum is to be represented by today's mathematics textbooks, efforts must be made to ensure that their content will not be treated superficially or ignored at will by the teachers. This is especially important in light of the current mathematics teacher shortage. To fill vacancies, many schools have assigned teachers with no formal mathematics training to teach mathematics. One wonders what type of mathematics is being taught to the students in these classes. A concerted effort is needed to attract more people with proper mathematics training to enter the field of education.

Text. Several areas of the mathematics text could be improved. First, the number of problems included in the exercises should be re-examined. Teachers should be given guidance on how to assign problems. This seems to be an area that merits further research in order to guide curriculum revision efforts. There also seems to be an overabundance of problems emphasizing the application of concepts rather than the understanding of the concepts. By utilizing problems which would develop a more conceptual understanding, the students' comprehension of concepts might be increased.

Second, the fundamental algebraic concepts should be introduced more gradually and integrated with other topics of the text. This would avoid overwhelming the students with too much too soon, allow for the learning of the concepts in more diverse and relevant contexts, and prevent the teachers from easily skipping over the material in their instruction.

Third, the chapter on analytical geometry seems to be isolated and places special emphasis on application of formulae. The students did not seem to be understand the derivations of the formulae and tended to forget or become confused with the formulae once they moved on to other chapters.

Problem Solving. To improve the performance of the lower achievers, the findings suggest that they should be introduced to rules and formulae which would expedite their problem-solving. However, these rules and formulae must be introduced only after the students have grasped the concepts, and in such a way that the students understand their derivations. Otherwise, as the study suggests, the students will easily forsake understanding for convenience.

Teaching Immigrant Students. Teachers of Chinese immigrant students, at least those from Southeast Asia, must place great emphasis on the understanding of mathematics concepts in their instruction. Since most of these students have been taught to apply rules and formulae in doing problems expediently in tests, they tend to project an impression that they are all high achievers. However, their ability to obtain good scores in examinations or tests does not imply that what they have learned matches the objectives of the U.S. mathematics curriculum. Teachers must be sensitive to the special needs of these students and tailor instruction to guide the students into the essence of the U.S. mathematics curriculum.

Other mathematics courses. Many of the lower achieving students who were placed in the Algebra I class were overwhelmed by the many abstract concepts and the "profound" way by which the basic number properties were explained. These students could benefit from a pre-algebra course where the fundamental number theory and properties are treated in depth but gradually. This course would help the students master the basics and facilitate their learning of the Algebra I curriculum.

VI. CASE STUDY 1: KUNG-MON CHANG

Kung-Mon Chang, an immigrant from Mainland China, came to the U.S. in November 1981 and entered Mission High School and the Algebra I class in the third month of the school year. His family had lived in the Four-District area of Canton, the homeland of most early Chinese immigrants, and was part of the large group that emigrated after the normalization of relation between the U.S. and the Peking government.

Kung-Mon came with his parents and an older brother. Both parents had been teachers in China, the father at a middle school and the mother at an elementary school. After arriving in the U.S., the father was working as a salesman in a grocery store and the mother as a seamstress in a sewing factory.

Kung-Mon had studied algebra for three years, since the seventh grade. It was taught in China as part of a six-year series in an integrated, middle-school math curriculum (which, according to Kung-Mon, was replaced by a segregated one similar to that of the U.S. in September 1981). In China he had attended a provincial "main-point" middle school, established after the cultural revolution to offer the higher achievers a more vigorous curriculum. Admission to these schools is by province-wide testing. In China, Kung-Mon's favorite subject at school was literature, but his mathematics was also considered very good.

PERCEPTION OF ALGEBRA I

Kung-Mon thought that the content of Algebra I was similar to that of courses taught in China. The course here was, however, easier, with slower pace of instruction, less homework, and less competitive examinations. He thought that he had learned all the contents of the textbook in China, and his objective of staying in this class was to learn the English (which he considered a very difficult but his most urgent task) without worrying about the mathematics. He said that he had difficulties with the concepts taught in the class because theywere usually associated with many English words. He thought that the instructional method was different from that in China, but he could not pinpoint the difference.

When asked about the first three chapters of the text, which he missed because he entered the class late, he said that he looked at the problems in the exercises, thought he could do them, and did not bothered with them anymore.

After a month in the class, Kung-Mon had become known as a high achiever in mathematics and became the central figures with the students sitting around him. He usually finished the seatwork assignments very fast and would help other students with their work or work on his assignments from other classrooms. He was not very attentive during instruction, and would sometimes engage in other activities while the teacher was lecturing. For example, he would be reading a book with the help of a dictionary. He said that since he knew the contents already, he would rather use the time to learn more English words.

His final examination score of B+ was disappointing to him. But overall, the teacher awarded him an A for the course, saying that he was one of the best mathematics achievers in the class.

TEST SCORES

Arithmetic Reasoning:	8/10
Piagetian:	50/72
General Ability Test:	Analogies 15/24 Computation 14/26 Classification 16/26
Course Grade:	1st semester/A- 2nd semester/A

WORD ASSOCIATION TEST

Because of his handicap in writing, Kung-Mon gave the responses to this test orally. The answers were tape-recorded and transcribed into the test booklet afterward by the researcher. Kung-Mon encountered no difficulties with the two examples of "mother" and "tiger." During the actual test, however, he did not associate the stimulus words/concepts with other mathematics concepts. Instead, he gave examples or explanations.

Examples:

INEQUALITY: $2 \neq 3$; $3 \neq 4$; $5 \neq 4$; $7 \neq 8$; ...., etc.

EXPONENT: 2^1, 1 is exponent; 2^2, 2 is exponent; 4^3, 3 is exponent; ...., etc.

He continued to gave this type of response, even though the researcher twice reminded him to try to think about more diverse responses. He also failed to gave any response to the concepts $\emptyset$ and VARIABLE. Kung-Mon commented later that an empty set consisted of no elements, so he could not gave any response. He also said that he had not encountered the concept VARIABLE before and was not sure what it was. Because of the inappropriateness of his responses, no further analysis was performed on his word association data.

SORTING TEST

Kung-Mon sorted twenty-seven of the twenty-eight concepts into nine groups. The concept IDENTITY was not sorted into any group. He said that he was not familiar with this concept because it was taught at the beginning of the school year prior to his arrival in the U.S. The nine groups are:

Group	Words/Concepts	Name of Group
(1)	Graph, +, —, X, +, <, >, ∩, √‾	Math symbols
(2)	Point, Plane, Coordinates, Line	Geometry
(3)	Commutative, Associative, Distributive	Properties
(4)	Rational Number, Polynomial, Equation	Unclear
(5)	Absolute Value, Exponent, Radicals	
(6)	Quadratic, Element	
(7)	Factor	
(8)	Set	
(9)	Slope	

PROBLEM SOLVING

Kung-Mon's problem-solving process followed, for the most part, to the three general steps suggested by Gagne (1980). However, the following are several special features of Kung-Mon's problem solving process:

(1) In the first step of the problem-solving process, reading and understanding the problem, Kung-Mon always introduced an additional step of reading the problem a second time in Chinese, and sometimes would ask the researcher for an explanation of certain English words.

(2) Kung-Mon tended to use rules or formulas he had learned in China. Many of these were not taught in the Algebra I class.

Problem I-12: Find the solution set for $-7x \leq 4$.

Kung-Mon: The method taught by the teacher and the textbook was to solve for x in the equation $-7x = 4$, and mark the solution (-4/7) on the number line; then take any point on either side of the point -4/7, test if it satisfies $-7x \leq 4$, and determine the solution set by means of the number line.

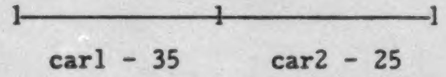
As soon as Hung-Mon saw the problem, he used the rule of transferring the coefficient (-7) of x to the right hand side of the inequality as the denominator and reversing the inequality sign because the coefficient was a negative number. The solution was arrived at immediately, in comparison to the lengthy process taught by the teacher and the book.

Problem I-10: Find the solution set: $|x - 1| = 3$.

Kung-Mon: The teacher and the textbook used the number line to arrive at the solution to problems involving absolute values. Kung-Mon, however, automatically divided up the problem into two, $(x-1) = 3$ and $(x-1) = -3$. He then found the numerical solution to each of the two problems. He never referred to the number line for the solution.

Problem II-17: Two cars start from the same point and travel in opposite directions at the rate of 25 and 35 miles per hour, respectively. In how many hours will they be 330 miles apart?

Kung-Mon: After reading the problem three times (several words at a time, first in English, then in Chinese), he proceeded to draw the following diagram to help him understand the problem:



The method of drawing a diagram for this type of word problem was not taught or demonstrated in the course (established from teacher interview), nor was Kung-Mon a frequent user of diagrams in problem solving. In fact, other examples showed that he seldom did so. He later commented that he had encountered this type of problem in China, and his mathematics teacher had shown how to use a diagram to solve the problem.

(3) Kung-Mon always attempted the problems with the rules he learned in China and would only resort to the new methods taught in the Algebra I class when his "old tricks" did not work.

Problem III-8: Graph the solution set $x^2 - 4 > 0$.

Kung-Mon: He solved the equation $x^2 - 4 = 0$ by factoring and arrived at the solutions of $x_1 = -2$ and $x_2 = 2$. He plotted the two points, -2 and 2, on the number line, but could not determine which parts of the number line represented the solution set. After examining the number line silently for a moment, he decided to use the rules taught in the class: if $ab > 0$, then either $a > 0$ and $b > 0$, or $a < 0$ and $b < 0$. By applying this rule, he arrived at the solution that $x < 2$ and $x > -2$.

Problem II-20: Factor $84x^2 + 135xy - 264y^2$

Kung-Mon: He wrote down

$$\begin{array}{cc} x & y \\ x & y \end{array}$$

and tried to put different combinations of factors of 84 and 264 as coefficients of x and y. However, because of the large number of combinations, he was not able to arrive at the correct answer. He then began the problem again using the method taught in the class of transforming the polynomial into a form with a perfect square as the first term. He wrote:

$$\begin{aligned} & 84x^2 + 135xy - 264y^2 \\ = \; & (1/84)[84(84x^2 + 135xy - 264y^2)] \\ = \; & (1/84)[(84x)^2 + 135(84x)y - 264(84)y^2] \end{aligned}$$

Then he proceeded to factor this quadratic form, though equally without success.

(4) Kung-Mon tended to associate alphabet variable (except for the three letters x, y, and z) with positive numerals.

Problem I-4: Under what conditions does $|a + b| = -(a + b)$?

Kung-Mon: He gave the incorrect answer of $a + b = 0$. He said that he arrived at this answer because he had associated the letter variables a and b with positive numbers.

Problem III-2: $(X^2)^{\frac{1}{2}} = ?$

Kung-Mon: $|X| \begin{matrix} x = a & (x > 0) \\ x = -a & (x < 0) \end{matrix}$

(5) Kung-Mon had difficulty doing the problems that required a proof instead of an answer or a solution set.

Problem II-13: The operation * is defined as: $a * b = 2a + b$
Is * commutative? Why?
Is * associative? Why?

Kung-Mon: He looked at this problem and could not figure out what to do. He had difficulty understanding what * is, and did not know how to prove the three basic properties.

Problem II-18: Is the following proof correct? Why?

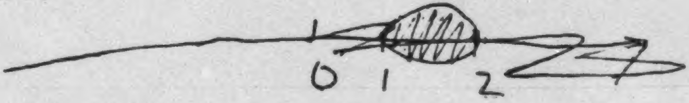
$$\begin{aligned} &\text{Let } x = 2 \text{ and } y = 1.\text{ Then}\\ x^2 - 2xy + y^2 &= x^2 - 2xy + y^2\\ x^2 - 2xy + y^2 &= y^2 - 2xy + x^2\\ (x - y)^2 &= (y - x)^2\\ (x - y) &= (y - x)\\ 2x &= 2y\\ x &= y\\ 2 &= 1 \end{aligned}$$

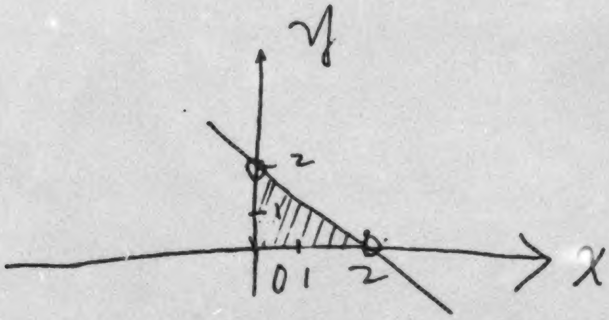
Kung-Mon: He did not know how to approach the problem. The researcher told him to check the series of equations to see if they were correct, but he could not detect the error. Finally, he substituted the x's and y's with the 2's and 1's to check if each of the equalities was correct. He found that $(x-y) \neq (y-x)$ and pointed out that was where the proof was incorrect. But he did not see that the inequality was due to the multi-root nature of $[(x-y)^2]^{\frac{1}{2}}$ and $[(y-x)^2]^{\frac{1}{2}}$.

(6) Kung-Mon had many difficulties with the two-dimensional graph. He could do the straightforward types of problems which asked him to plot linear or quadratic equations but he was unable to visualize relations among large numbers of points on a graph. Towards the end of the school year, he seemed to improve and was able to do some of the two-dimensional graph problems.

Problem II-5: Graph all points (x, y) such that x and y are both between 1 and 2.

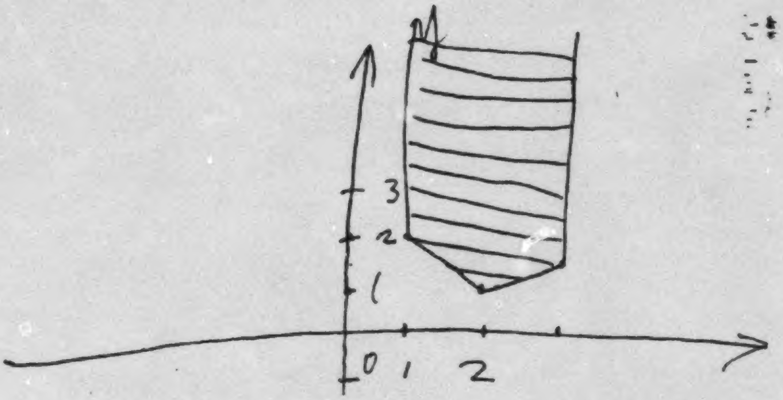
Kung-Mon:





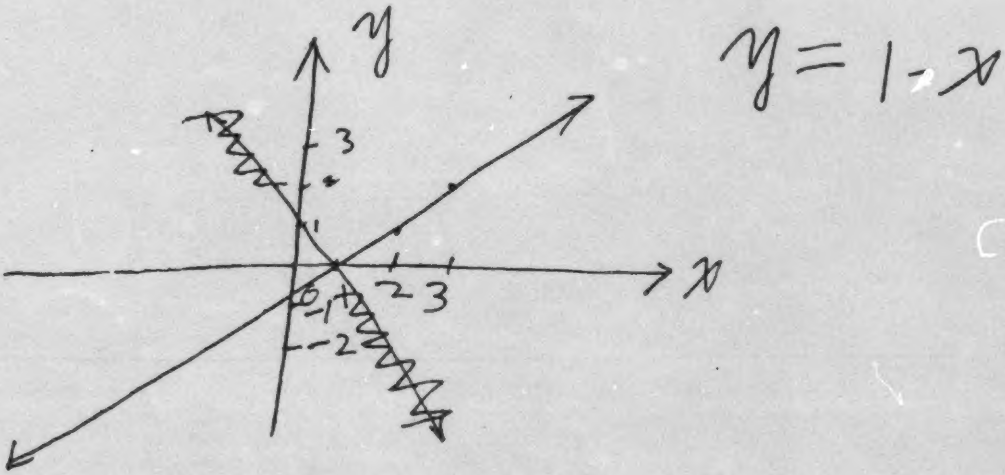
Problem II-8: U = [1, 2, 3], graph [(x,y): y>2/x].

Kung-Mon:



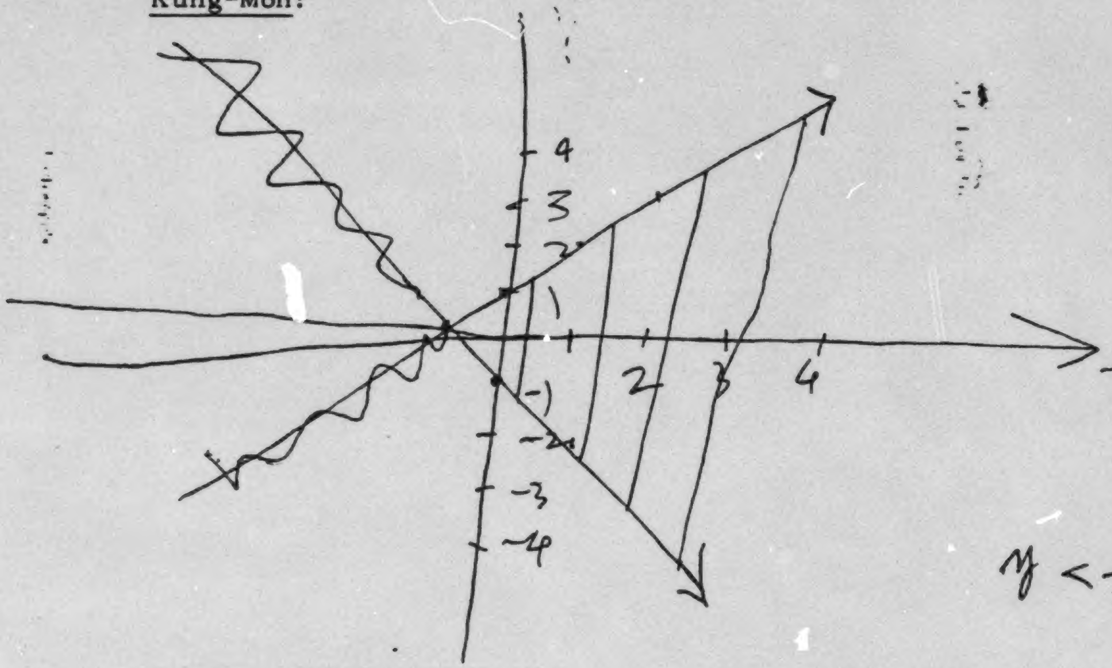
Problem II-10: Graph the solution set of x + y = 1.

Kung-Mon: He transformed the equation to y = 1 - x, and derived two points, (1,0) and (0,1). He plotted a straight line through these two points (the correct solution). But after looking at the line for a while, he deleted it and drew another, which was incorrect.



Problem II-12: Graph $[(x, y) : y < |x + 1|]$

Kung-Mon:

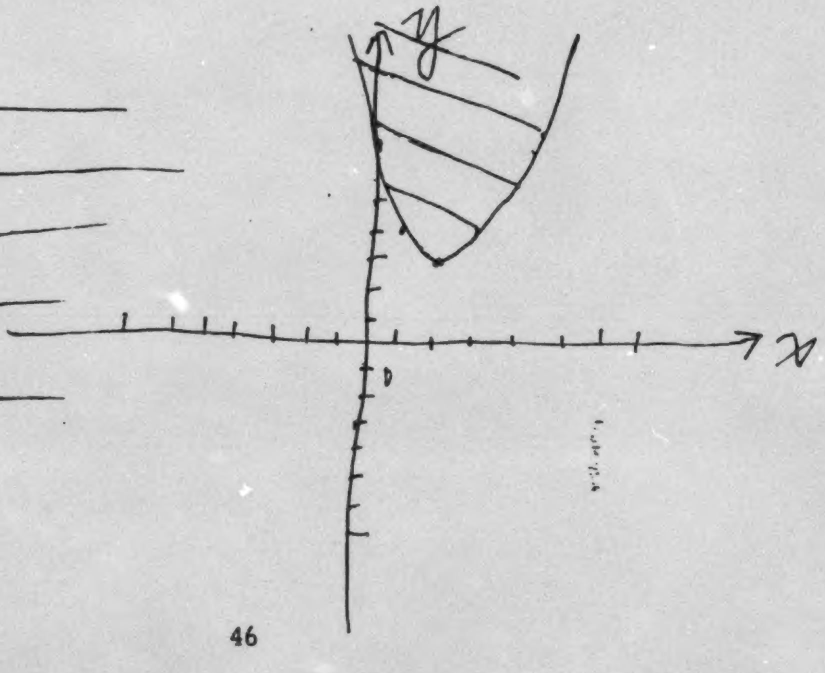


$y < -(x+1)$

Problem III-10: Graph $y \geq (x - 2)^2 + 3$

Kung-Mon: He wrote down a series of number pairs, plotted the graph and shaded in the solution set.

x	y
1	4
2	3
3	4
-1	12
4	7



(7) Kung-Mon is very good at the basic algebraic operations.

Problem III-5: Simplify $\frac{2}{a-2} + \frac{1}{a^2-4} + \frac{3}{a+2}$

Kung-Mon: He was able to go through the problem in the following concise steps.

$$= \frac{2}{(a-2)}(a+2)(a-2) + \frac{1}{(a+2)(a-2)}(a+2)(a-2) + \frac{3}{(a+2)}(a+2)(a-2)$$

$$= 2a+4+1+3a-6$$

$$= 5a-1$$

(8) Kung-Mon seldom checked his answer, especially when there did not seem to be any difficulty in arriving at the solution(s).

Problem III-6: $x = (x-2)^{\frac{1}{2}} + 4$, $x = ?$

Kung-Mon: First he transformed the equation to $(x-2)^{\frac{1}{2}} = (4-x)$, an incorrect step which did not affect the solution to the problem. He then squared both sides of the equation, transformed the equation into the $x^2 + ax + b = 0$ form, factored the quadratic equation, and obtained the answers of $x1=3$ and $x2=6$. He was not aware that there is only one root to the equation and he never bothered to substitute the answers back into the equation to check for correctness.

CONCLUSION

Kung-Mon was a high mathematics achiever in China. He had had three year of algebra before coming to the U.S., and had been introduced, though with different emphasis, to most of the curriculum contents of the Algebra I course. He had a very good command of the manipulation of basic algebraic operations, and was in good control of a set of rules and formulae to solve algebraic problems. However, his knowledge in algebra seemed to have interfered with his learning of the contents taught in the class. Because he could do many of the problems in the textbook by applying his rules and formulae, he often ignored the classroom instruction during which the instructor, Mr. Wong, would give lengthy explanations to reinforce the students' understanding of the algebraic properties and structures.

Kung-Mon was especially weak in the problems involving proofs and graphs. Perhaps these two areas were not taught in the first three years of the algebraic curriculum in China. However, since these two types of problems also required more abstract reasoning involving algebraic concepts, the problem might have stemmed from Kung-Mon's lacking a good understanding of the algebraic properties and structures. This lack was corroborated by his performance on the Word Association Test, where he was only able to give examples and not other mathematical concepts in response to the stimulus concepts.

VII. CASE STUDY 2: WINNIE CHENG

Winnie Cheng came to the United States in the late summer of 1981 from Hong Kong. Of upper-middle class background, she and a younger sister were staying in a relative's home in San Francisco. Her parents remained in Hong Kong, where her father was a police inspector and her mother a housewife. Winnie and her sister came to the United States for a better educational opportunity, but ironically, Winnie had attended one of Hong Kong's most prestigious middle schools, known for its students' academic achievement, and she had learned algebra since the fifth grade as part of the integrated school mathematics curriculum. She said that she was among the school's better achievers and that mathematics was one her best subjects. Her main interest, however, was in biology, and she wanted to become a doctor.

PERCEPTION OF ALGEBRA I

Winnie thought that the U.S. mathematics curriculum was different from that of Hong Kong. It treated the different properties of the number system in more depth and gave everything a name, and thus making a formal system. She had learned all those properties before, but now she also learned their names. Winnie thought that the Algebra I course was easy compared to the mathematics course in Hong Kong, where they taught more in a day and gave more homework. She did not encounter much difficulty except with several concepts which she had not encountered in Hong Kong, such as absolute value and graphs. But even these, she said, she developed a good understanding of after a while. However, since mathematics was taught entirely in English at the school she had attended in Hong Kong, she was not quite accustomed to the Chinese mathematics terms use in the bilingual Algebra I course.

She enjoyed Mr. Wong's teaching and developed a good rapport with him, often going to ask him questions during the lunch hour.

TEST SCORES

Arithmetic Reasoning:	7/10
Piagetian:	60/72
General Ability:	Analogies 16/24 Computation 20/26 Classification 26/26
Course Grade:	1st semester/A 2nd semester/A

WORD ASSOCIATION TEST

Winnie wrote down mathematics words/concepts to six of the twenty-two stimulus words/concepts: NATURAL NUMBER, ELEMENT, RATIONAL NUMBER, ADDITION, PERCENTAGE, MULTIPLICATION, and EMPTY SET. To the other sixteen, she gave examples or explanatory notes.

Examples:

SUBSET: $[1,2] \subseteq [1,2,3]$; $\subseteq$; $A \subseteq B$

COMMUTATIVE PROPERTY: $a + b = b + a$; $a \cdot b = b \cdot a$

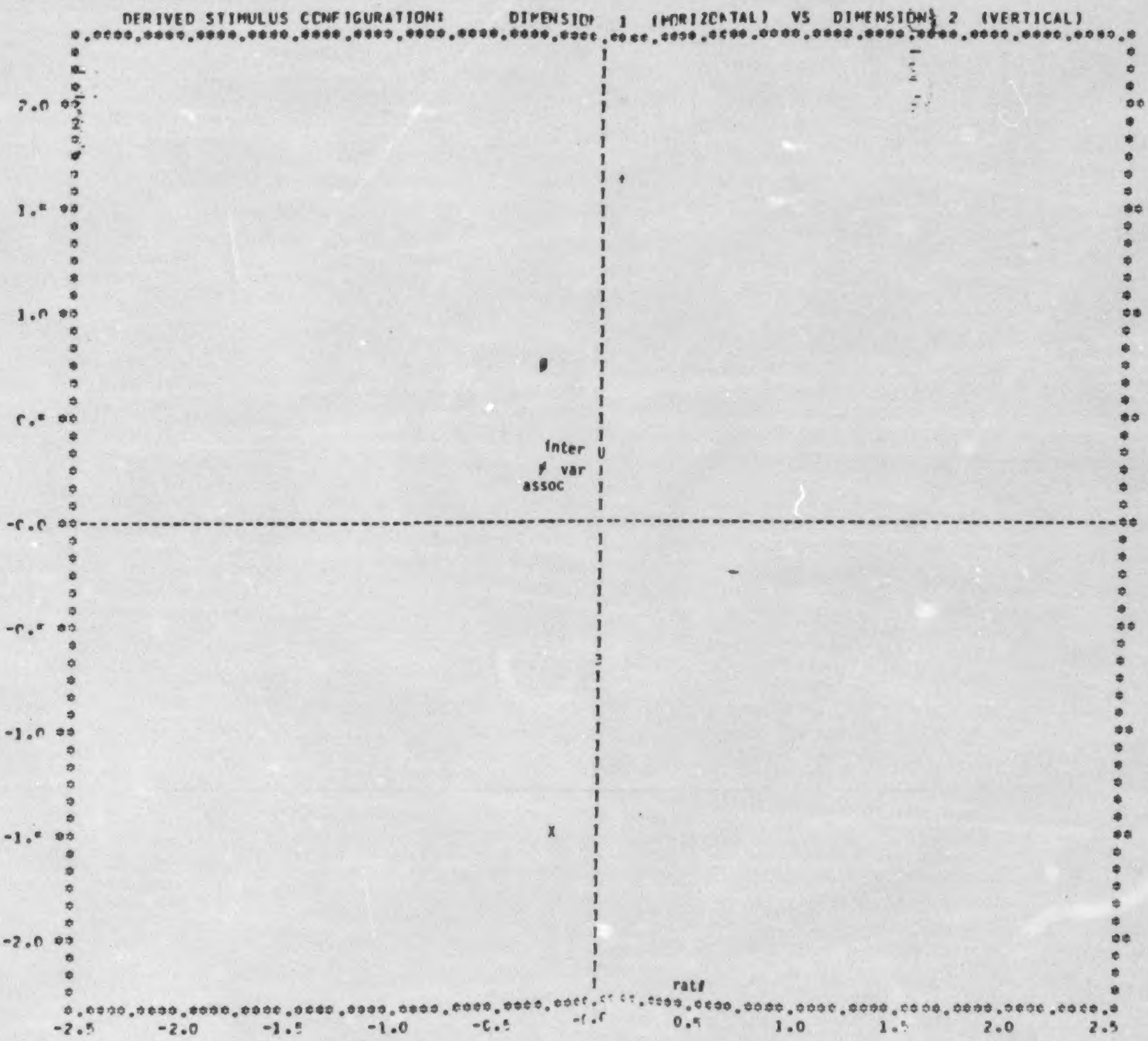
EXPONENT: x^2; The power of n is equal to x^n; $x \cdot x = x^2$

The following are the RC matrix derived from Winnie's response and its corresponding two-dimensional graphical representation.

TABLE 1: RC matrix of Winnie Cheng's Word Association Test responses.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1 Set	1.0																					
2 1:1		1.0																				
3 natural #	.207		1.0																			
4 integer				1.0																		
5 subset					1.0																	
6 commutative						1.0																
7 associative							1.0															
8 ÷								1.0														
9 ≠									1.0													
10 element	.544		.093							1.0												
11 rational #	.222										1.0											
12 U												1.0										
13 +	.044		.093							.019	.074		1.0									
14 counting #	.276		.692							.074	.370		.093	1.0								
15 distributive															1.0							
16 –																1.0						
17 identity																	1.0					
18 exponent																		1.0				
19 ∩																			1.0			
20 X								.138												1.0		
21 ∅	.138																				1.0	
22 variable																						1.0

FIGURE 1: Two-dimensional scaling solution of Winnie Cheng's RC matrix.



SORTING TEST

Winnie sorted the twenty-eight concepts into six groups:

Group	Words/Concepts	Name of Group
(1)	Set, Element, ∩	Set Theory
(2)	Graph, Coordinate, Line, Point, Plane, Slope	Geometry
(3)	$\sqrt{\ }$, Radical, Exponent	Power
(4)	Associative, Distributive, Commutative, Identity	Properties
(5)	Factor, Quadratic, Polynomial, Equation	Polynomials
(6)	Absolute Value, Rational Number, $<$, $>$, ÷, +, −, X	Number system

PROBLEM SOLVING

Winnie was rated by the teacher as a high achiever in the Algebra I class. She was diligent and attentive, and always completed her homework assignments on time. Often she would read the textbook on her own to get another look at the contents taught in class, or to learn things skipped over by the teachers. She was transferred out of Mr. Wong's bilingual Algebra I class to a regular class during the second semester because the school counselor thought that she did not need the bilingual instruction and that her space could be used for another new immigrant student. Winnie said she liked Mr. Wong better than her new teacher, Ms. Hall, whose teaching fashion resembled that of her teachers in Hong Kong; they emphasized obtaining speedy correct answers, but not understanding. Ms. Hall skipped many sections of the text which dealt with graphs, and spent much of the semester on techniques of factoring polynomials.

The following are features of Winnie's problem-solving process:

(1) While very good at problem solving, Winnie did not know how to write proofs.

Problem I-3: True or false? Why?
If $a > 0$, $b < 0$ and $|a| > |b|$, then $a + b = -(|a| - |b|)$.

Winnie: After examining the problem for a little while, she substituted a and b with actual numbers (2 and 1) and gave the answer "false." The researcher asked her why. She said that she did not know, but she was sure of her answer.

Problem II-13: The operation * is defined as: $a * b = 2a + b$.
Is * commutative? Why?
Is * associative? Why?

Winnie: She said that she had seen this type of problem before but did not know how to do it. After looking at it for a little while, she gave up.

Problem II-18: Is the following proof correct? Why?
Let $x = 2$ and $y = 1$. Then

$$\begin{aligned} x^2 - 2xy + y^2 &= x^2 - 2xy + y^2 \\ x^2 - 2xy + y^2 &= y^2 - 2xy + x^2 \\ (x - y)^2 &= (y - x)^2 \\ (x - y) &= (y - x) \\ 2x &= 2y \\ x &= y \\ 2 &= 1 \end{aligned}$$

Winnie: She examined the problem and asked how 2 could be equal to 1. The researcher told her that was what the problem wanted her to explain. She said that she knew that it could not be correct, but she did not know why.

(2) Of the eight target students, only Winnie could solve the following problem without any difficulties.

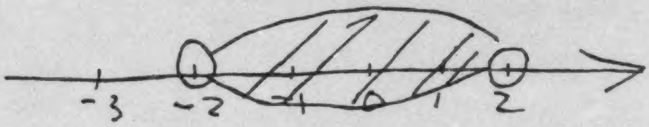
Problem I-4: Under what conditions does $|a + b| = -(a + b)$?

Winnie: She saw at once that the equation was equivalent to $|c| = -(c)$. Therefore the condition was $(a + b) < 0$. She never did bother with the individual value of a or b like all the other students.

(3) Winnie completed many of the problems very quickly by applying many rules and formulae. Some were taught in the Algebra I class and some she had learned in Hong Kong, but many she had deduced by herself from doing the exercises. From her classwork or homework, she would discover some rules or formulae which provided her with shortcuts to arrive at the solution, and she would then bring these at lunch hour to Mr. Wong, who verified for her their correctness.

Problem I-11: Graph $|x| < 2$.

Winnie: She saw at once that for $|x| = 2$, x is equal to -2 or 2. She then drew the following graph immediately.



When asked how she arrived at this graph, she answered that if it is the 'smaller than' inequality sign, the solution set is between the two points and if it is the 'bigger than' inequality sign, the solution set is on the two sides of the two points. This rule she deduced herself and was confirmed by Mr. Wong to be correct.

Problem I-12: Find the solution set $-7x \leqslant 4$.

Winnie: She wrote down the following steps:

$$-7x \leqslant 4$$
$$-4 \leqslant 7x$$
$$-4/7 \leqslant x$$

She indicated that she went from the first step to the second by using a rule she deduced: switch the positions of the values (but not the negative sign) on the two sides of the inequality sign. This she had deduced through her exercises.

(4) Winnie's application of rules and formulae in problem solving did not always bring success, especially when she tried to apply those she had not used for a while. She would memorize the inequality signs or the orders of the variables wrongly.

Problem II-14: Find the coordinates of the midpoint of the segment $\overline{(2, 3), (4, 9)}$.

Winnie: Instead of the formula of $(x_1 + x_2)/2$, $(y_1 + y_2)/2$, she memorized the formula as $(x_1 - x_2)/2$, $(y_1 - y_2)/2$ and got the incorrect answer of $(-1, -3)$.

Problem II-12: Graph: $[(x, y): y < |x+1|]$.

Winnie: She applied the rule of separating $y < |x+1|$ into two inequalities but had the negative signs confused:

$$y < x + 1$$
$$-y < x + 1$$

(5) Except for the straightforward type, Winnie could not do the two-dimensional graph problems.

Problem II-10: Graph the solution set of $x + y = 1$.

Winnie: She transformed the equation to $x = 1 - y$ (x-form!), calculated several number pairs, plotted the points on the graph, and drew the straight line through these points.

Problem II-5: Graph all points (x, y) such that x and y are both between 1 and 2.

Problem II-8: $U = [1, 2, 3]$, graph $[(x, y): Y > 2/x)\}$.

Problem II-12: Graph: $[(x, y): y < |x + 1|]$.

Problem II-16: Graph: $[(x, y: |x - 1| \leq 1]$ $[(x, y): y \leq |x|]$.

Winnie: She was not able to do any of these problems. She either said that she could not remember the formulae or that her second semester teacher, Ms. Hall, did not teach them any thing on graphs and she had forgotten what Mr. Wong taught in the first semester.

(6) Throughout the three phases of testing, Winnie never checked the correctness of her answers. When asked, she commented that it was a habit developed in Hong Kong, where they were taught to get the answers to problems as fast as possible so they could perform well on examinations, which usually consisted of timed tests. The emphasis was on speed and result, but not process. Once finished with a problem, the students moved right on to another and would not "waste" time in checking the answers. They would check their answers only if there was time left after completing the whole test.

(7) Because of the emphasis on speed, Winnie did not use any trial-and-error approach to problem solving. She would try to memorize formulae or similar problems she had done before, but if these were not quickly recalled, she would easily give up and go on to the next problem.

CONCLUSION

Winnie was a very good mathematics achiever and was learning the Algebra I contents very well. She understood the difference in emphasis of the mathematics curriculum in Hong Kong and the U.S. and focused on understanding algebraic concepts. Her progress in Mr. Wong's class was excellent. She regretted being transferred during the second semester into Ms. Hall's class, where the emphasis shifted back to manipulating operations.

VIII. CASE STUDY 3: ROSE HUYEN

Rose came from Vietnam. Her parents had tried to leave Saigon since 1977 by boat, but the cost was always too high. Just when they were about to give up, their U.S. immigrant visas were approved, and they flew to San Francisco in 1980. Rose now lives with her parents, two younger brothers and a younger sister. Both parents were attending adult school. In Saigon, her father was a merchant and her mother a housewife.

Rose has attended the tenth grade in a private Chinese school before coming to the U.S. She said that she had learned some algebra in her secondary grades. During her first year in San Francisco, Rose attended the Newcomers' High School and took a general math class. She transferred to Mission High School in September 1982 as a tenth-grade student.

The teacher, Mr. Wong, thought Rose was a very hard-working student, though she was not very good in mathematics. In the second semester, she was transferred into Ms. Hall's Algebra I class because the counselor did not think that Rose required the bilingual instruction of Mr. Wong's class (though her younger brother was not transferred).

PERCEPTION OF ALGEBRA I

Rose thought that Mr. Wong's class was difficult. She usually did not understand the teacher's explanation in the classroom. Instead, she had to read the textbook, with the aid of a dictionary, at home, to try to follow the lessons. The Algebra I course was confusing, but she could not pinpoint where and how. The contents were new and overwhelming. She said that Mr. Wong's instruction was very clear, but that he always explained things in a very profound way. For example, he had to spend half an hour just to explain that $a(b + c) = ab + ac$, which was an "obvious fact."

She said that she could usually do the problems in the textbook by following the examples, but she admitted that she did not understand many of the processes.

She said that the second semester algebra class taught by Ms. Hall was easier, and she was not as confused. She also received a better grade from Ms. Hall, who considered her one of the best students.

TEST SCORES

Arithmetic Reasoning:	6/10
Piagetian:	46/72
General Ability:	Analogies 15/24 Computation 12/26 Classification 17/26
Course Grade:	1st semester/C 2nd semester/B+

WORD ASSOCIATION TEST

Except for the first stimulus word/concept, SET, to which Rose wrote down "empty," "empty set," "subset," and "absolute value," she failed to give appropriate responses. Instead, she wrote down lists of examples.

Examples:

INTEGER: 0; 1; 2; ···; 64

COMMUTATIVE PROPERTY: $a + b = b + a$; $c + d = d + c$; $d + f = f + d$; ...; $33 \times 4 = 4 \times 33$

Because her responses were not those called for by the test, no further analysis was performed on the data.

SORTING TEST

Rose sorted the twenty-eight concepts into nine groups:

Group	Words/Concepts	Name of Group
(1)	U, Quadratic, Graph, Coordinate, Line, Slope, Absolute Value	Graph & functions
(2)	+, +, -, X	Basic operations
(3)	Associative, Distributive, Commutative, Identity	Properties
(4)	$>$, $<$	Inequalities
(5)	Rational Number, Set, Element	Unclear
(6)	$\sqrt{\ }$, Radical, Polynomial, Equation	
(7)	Point, Factor	
(8)	Plane	
(9)	Exponent	

PROBLEM SOLVING

Rose refused to use the "think-aloud" method to do the problems. She said that she could not think and talk at the same time. Despite continuous effort from the researcher to elicit "talks" from her during the three problem-solving sessions, she did not give any oral responses. Nevertheless, Rose did answer questions on her problem--solving processes when she completed all the problems in a session. The following analysis of Rose's problem-solving features was based mainly on the researcher's observations, the accompanying field notes, and Rose's explanations.

(1) Rose usually would not write out a sequence of deductions and proofs when solving a problem. Instead, she liked to do all the calculations or deductions "in her head." She usually took a very long time to do a problem because of her "intense thinking," and she always

arrived at incorrect answers because of simple mistakes committed in carrying lengthy operations in her head. The following example is from a page of her test booklet. She stared at the problem and wrote down the answer after thirteen minutes.

Problem I-9:

$$\text{If } a > b \text{ and } k < 0\text{, then } ka \overset{>}{\underline{\quad ? \quad}} kb.$$

Problem II-9: The statement, "A certain number, f, increased by twice another number, n, is equal to 30," can be written as:

Rose: She wrote down the correct mathematics sentence ($f + 2n = 30$) and looked at it for over ten minutes. Then she wrote down an incorrect sentence ($f + 2n = 30$), looked at it for two minutes, crossed out the incorrect answer and went on to the next problem.

After the first problem-solving session, the researcher told her that writing down the steps of her thinking might help her speed and accuracy. During the last problem solving session at the end of the school year, she was writing down more of her problem solving steps.

(2) Throughout the problem-solving sessions, Rose tended to use the methods she had learned in Vietnam. She said that the methods taught in the Algebra I class were too lengthy and cumbersome, while she could usually arrive at an answer more directly by the methods she had learned earlier. She admitted though, that the method taught by the Algebra I teacher was clearer. She said that as long as she could get the answer, why worry about how you got it. For example, many of the problems involving inequalities were supposed to be solved by using the number line, but she never used it.

Problem I-10: Find the solution set $|x - 1| = 3$.

Rose: She did not use the number line. Instead, she wrote down $x = [\qquad\quad]$, and then tried to fill in the numbers, using the trial-and-error method.

In the next two examples, Rose used the trial-and-error method instead of those taught in the class to solve the problems.

Problem II-20: Factor: $84x^2 + 135xy - 264y^2$

Rose: She took out 3 as a factor at once and wrote

$$\begin{aligned} &= 3(28x^2 + 45xy - 88y^2) \\ &= 3(\ \ x \quad y)(\ \ x \quad y) \\ &= 3(\ \ x - y)(\ \ x + y) \end{aligned}$$

Next, she tried putting different combinations of factors of 28 and 88 as coefficients of x and y, and eventually arrived at the answer $3(7x - 8y)(4x + 11y)$.

Problem III-11: Find two integers whose sum is 18 and whose product is 72.

Rose: She wrote $m + n = 18$
$m \cdot n = 72$

Then she used the trial-and-error method and got the correct answers of 6 and 12.

(3) She had difficulties doing the problems involving two-dimensional graphs. She said that she could do those problems when she was in Mr. Wong's class, but since transferring to Ms. Hall's class in the second semester, she had not seen graphs at all and could not remember how to do those problems.

Problem II-5: Graph all points (x, y) such that x and y are both between 1 and 2.

Rose: She looked at the problem for five minutes, gave up and went to another page.

Problem II-16: Graph $[(x, y) : |x - 1| \leqslant 1] \cap [(x, y) : y \leqslant |x|]$.

Rose: She drew $|x - 1| \leqslant 1$ on the x-axis of a two dimensional graph (an incorrect answer). This took three minutes. Then she drew the $y \leq |x|$ graph using number pairs and inspection. This took her over ten minutes. After that she did not know how to put the two graphs together.

(4) Rose was not able to do the problems that required proofs or creative application of principles or concepts learned in the class.

Problem II-13: The operation * is defined as : $a * b = 2a + b$.
Is * commutative? Why?
Is * associative? Why?

Rose: She looked at the problem for two minutes, and went on to the next one.

Problem II-18: Is the following proof correct? Why?

Let $x = 2$ and $y = 1$. Then

$$\begin{aligned} x^2 - 2xy + y^2 &= x^2 - 2xy + y^2 \\ x^2 - 2xy + y^2 &= y^2 - 2xy + x^2 \\ (x - y)^2 &= (y - x)^2 \\ (x - y) &= (y - x) \\ 2x &= 2y \\ x &= y \\ 2 &= 1 \end{aligned}$$

Rose: She looked at the problem for serveral minutes, and did not know how to approach the problem. Then she substituted the numbers 2 and 1 in each equation and found that $(x - y) \neq (y - x)$. She pointed to that equation and said that that was incorrect.

Problem II-7: If $(x + 7)^2 - 1 = 0$, $x = ?$

Rose: Instead of applying the identity, $(a^2 - b^2) = (a+b)(a-b)$, she multiplied out $(x + 7)^2$, subtracted 1 from 49, and then solved the equation $x^2 + 14x + 48 = 0$.

Problem II-15: $6987 \times 7013 = ?$

Rose: She looked at the problem for a little while, decided that it was to lengthy to multiply out the two numbers and gave up.

(5) At the end of school year, Rose was making errors on problems involving cancelling factors on the numerator and denominator of fractions.

Problem III-5: Simplify $\frac{2}{a-2} + \frac{1}{a^2-4} + \frac{3}{a+2}$.

Rose: $= \frac{2}{a-2} + \frac{1}{(a+2)(a-2)} + \frac{3}{a+2}$

$$= \frac{2(a+2) + 1 + 3(a-2)}{(a+2)(a-2)} = 6$$

$$= \frac{2a + 4 + 1 + 3a - 6}{(a+2)(a-2)}$$

$$= \frac{5a - 1}{(a+2)(a-2)}$$

Problem III-14:
Find the roots of $\frac{3}{2x-6} - \frac{2x-3}{2x^2-5x-3} + \frac{1}{2x+1} = 0$

Rose: $\frac{3}{2(x-3)} - \frac{2x-3}{(2x+1)(x-3)} + \frac{1}{2x+1} = 0$

$$\frac{3(2x+1) - (2x-3)2 + 2(x-3)}{2(x-3)(2x+1)} = 0$$

$$3 - 4x + 6 + 2 = 0$$

$$-4x + 11 = 0$$

$$4x = \sqrt{11}$$

$$4x = \pm\sqrt{11}$$

$$x = \frac{\pm\sqrt{11}}{4}$$

CONCLUSION

Despite her hard work, Rose was not very successful in learning the Algebra I contents, as was indicated by her performance in the problem-solving sessions. She was confused by the emphasis on the understanding of the algebraic concepts in Mr. Wong's class. Perhaps this was due to the interference of the mathematics she had learned in Vietnam, which emphasized the application of rules and formulae to obtain answers. Rose, however, was more in tune during the second semester with Ms. Hall's Algebra I course, which was similar in style to the mathematics curriculum in Saigon. Ms. Hall considered Rose to be a good mathematics student and awarded her a grade of B+ for the course (though her performance during the last problem-solving session was disappointing).

IX. CASE STUDY 4: STANLEY KWOK

Stanley Kwok came to the U.S. with his parents, an older brother, and a younger sister from the city of Guangzhou (Canton) in 1979. They came via Hong Kong, where they stopped over for half a year, before arriving in New Orleans. After six months there, the family moved to San Francisco. Stanley's parents were factory workers before emigrating to the U.S. At the time of this study, his father was unemployed and his mother worked in a garment factory.

Before leaving China, Stanley had finished three months of sixth grade, during which algebra has been taught as part of an integrated math curriculum. He did not attend school during his stay in Hong Kong, but entered seventh grade in New Orleans and learned general math. Upon arrival in San Francisco half a year later, he entered ninth grade at Mission High School and took another year of general math.

Mr. Wong considered him a below-average mathematics achiever. He said that Stanley was alienated from the school and did badly last year, but that he was working diligently and taking interest in his school work this year.

PERCEPTION OF ALGEBRA I

Stanley thought that the Algebra I course was very difficult. The many terminologies, symbols, and signs made the contents confusing, and the many English words used with them made them hard to understand. There were still concepts (e.g. rational, irrational number, counting number, etc.) he did not understand at the end of the school year.

He thought that Mr. Wong was going at too fast a pace. Mr. Wong devoted most of the class to explaining concepts leaving very little time for classwork to reinforce what was taught. Stanley usually forgot what was taught when he went home and had to read the examples (not the text) in the book to refresh his memory. He said that he would also go ask Mr. Wong questions during the lunch hour. He wished the teacher would go more slowly and give more classwork to help students remember and understand.

Stanley found that the pace especially fast during the latter half of the first semester and the first half of the second semester, and felt he was completely lost. But Mr. Wong spent the last month of the school year on review, which allowed him to relearned the contents he had not understood. (He thought that was a reason he performed better at the final examination.) Stanley also commented that since the teacher did not check the homework personally, he would not know what he had done correctly. (Mr. Wong usually wrote the answers on the board and asked the students to check their own answers. But this, to Stanley, was not the same as the teacher checking the procedures and answers directly, which would give him more confidence.)

TEST SCORES

Arithmetic Reasoning:	6/10
Piagetian:	44/72
General Ability Test:	Analogies 8/25 Computation 14/26 Classification 20/26
Course Grade:	1st semester/D+ 2nd semester/C+

WORD ASSOCIATION TEST

Stanley wrote most of his test responses in Chinese. He said that he could not remember the spellings of the mathematical words. Except for one, all his responses were examples or explanations of the stimulus words/concepts. Because the responses were not those solicited by the test, no further analysis on these data was conducted.

Examples:

NATURAL NUMBER: not unusual; in order.

ASSOCIATIVITY: together; associate; not separate; linked.

X: multiplication; multiplication sign; multiply; multiply one number with another.

The only exception was for the stimulus word/concept "∅," to which he gave "inequality" as one of his responses.

SORTING TEST

Stanley sorted the twenty-eight mathematics words/concepts into ten groups:

Group	Words/Concepts	Name of Group
(1)	Set, Element	Set Theory
(2)	Identity, Associative, Distributive, Commutative	Properties
(3)	Polynomial, Equation, Factor	Polynomial
(4)	+, -, x, ÷	Operations
(5)	Slope, Coordinates, Line, Graph	Geometry
(6)	Exponent, Quadratic	Unclear
(7)	Radical, $\sqrt{\ }$, Rational Number	
(8)	Absolute Value, $\cap$, $<$, $>$	
(9)	Point	
(10)	Plane	

PROBLEM SOLVING

Stanley Kwok was not a high mathematics achiever. However, the teacher described him as adjusting well both in and out of school. He worked very hard and was progressing in the class as expected. Although quite confused by the algebra curriculum in the first half of the school year, by its end, he had managed to get a grip on the subject through his hard work (he said), and was able to perform quite well in the last problem-solving session. The following are his problem-solving characteristics:

(1) Stanley could not do any problem that did not ask for specific answers. These included problems that asked for proofs, explanations, or abstract solution sets.

Problem I-3: True or false? Why?

If $a > 0$, $b < 0$ and $|a| > |b|$, then $a + b = -(|a| - |b|)$.

Stanley: He read the problem but told the researcher he did not understand what he was supposed to do. The researcher explained the problem to him in Chinese. Stanley looked at the problem again and gave the correct answer, but could not explain his reasoning.

Problem I-4: Under what conditions does $|a + b| = -(a + b)$?

Stanley: He read the problem, did not know what to do, and told the researcher that he could not understand the English. The researcher translated the problem to him in Chinese, but he still did not know what to do.

Problem II-18: Is the following proof correct? Why?

Let $x = 2$ and $y = 1$. Then,

$$\begin{aligned} x^2 - 2xy + y^2 &= x^2 - 2xy + y^2 \\ x^2 - 2xy + y^2 &= y^2 - 2xy + x^2 \\ (x - y)^2 &= (y - x)^2 \\ (x - y) &= (y - x) \\ 2x &= 2y \\ x &= y \\ 2 &= 1 \end{aligned}$$

Stanley: He substituted the 2 and 1 in the first equation but did not know what to do after that.

(2) Stanley also used rules and formulae which were not taught in the class to do the problems. He said that he learned them from his classmates. Since the rules and formulae could lead to answers more expediently, he preferred them to the methods taught in the class.

Problem I-10: Find the solution set $|x - 1| = 3$.

Stanley: He changed the equation to $(x - 1) = \pm 3$ and arrived at the solution set (4, -2).

Problem I-11: Graph $|x| < 2$.

Stanley: He applied the ± rule again, obtained the two points (2, -2), and drew the answer on the number line.

For the problems on coordinate geometry, he was very dependent on rules and formulae. When he forgot them, he would just give up instead of trying to derive them. He could not do any of the following problems because he said that he had forgotten the formulae.

Problem II-3: What is the slope of the line: $3x - 5y = 15$.

Problem II-14: Find the coordinates of the midpoint of the segment (2,3), (4, 9).

Problem III-4: Find an equation of the line which contains the points (0,2) and (-1,3).

(3) During the second problem-solving session, Stanley encountered many difficulties with graphs and could not do any of the two-dimensional graph problems.

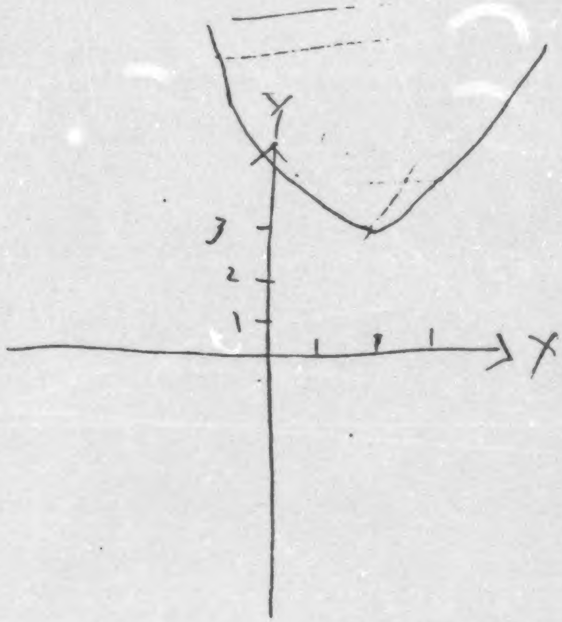
Problem II-10: Graph the solution set of $x + y = 1$.

Stanley: He drew the x-y coordinate plane at once and wrote down the number pairs for the equation. After that, he could not plot the points on the graph. After the researcher showed him where the points should be, he took the points, instead of a line passing through them, as the solution set.

However, in the third problem-solving session, Stanley made an impressive improvement and was able to do the two-dimensional graph problem quite proficiently.

Problem III-10: Graph $y \geqslant (x - 2)^2 + 3$.

Stanley: He saw the inequality and drew the following graph at once.



(4) Stanley could only do the straightforward type of problems which had been taught in the class. Any variation made a problem too difficult for him.

Problem II-4: If $k^3 + 2k^2 - 24k = 0$, then $k = ?$

Stanley: He looked at the problem and told the researcher he did not know how to do a problem with a x^3 term. The researcher told him that he could pull k out as a common factor, but he still could not do the problem.

CONCLUSION

The Algebra I course was overwhelming for Stanley. The pace of instruction was too fast and the abundance of algebraic concepts during the first semester was more than he could absorb. It was toward the end of the second semester, when the contents progressed to techniques of factoring, that he became more comfortable. The last month of review in the class also gave him opportunities to relearn many concepts he did not understand, and thus he made up much lost ground and eventually received a C for the course. Mr. Wong, the teacher, was very impressed by his progress and complimented Stanley's effort and diligence. However, Stanley said that he still did not understand many of the algebraic concepts taught during the first semester and wished he could have learned the course content at a slower pace.

X. CASE STUDY 5: CHUNG-KWONG LEE

Chung-Kwong Lee emigrated from Hong Kong to the United States in November 1981. Born and raised in Hong Kong, he had attended school there from kindergarten through nineth grade, when he dropped out because he had lost interest and was doing badly. Then he worked for two years before coming to the United States. He came with his parents; his father had been a carpenter in Hong Kong and his mother a housewife.

Chung-Kwong had one year of algebra in the nineth grade. He insisted that he was not too intelligent, and had not enjoyed schooling in Hong Kong. He also said that he had forgotten everything during his two years of working. He entered Mission High School and the algebra class in November, when the class was into its second month of instruction.

PERCEPTION OF ALGEBRA I

Chung-Kwong had a very low self-concept in mathematics. He kept telling the researcher and others that he was not good at algebra and not made out for school. He found the pace of instruction of the Algebra I class too fast, and could not understand the materials. He often asked his best friend, Kung-Mon Chang (another target student), who sat in front of him, for assistance. To learn the contents taught before he entered the class, he said that he read the chapters in the text and asked for help from other students who were better than he. As the school year progressed, he gradually remembered the mathematics he had learned in Hong Kong.

Chung-Kwong thought that the main difference between the Algebra I course and the course he had had in Hong Kong was in terminology, but the teachers were also very different. Mr. Wong was much better, he thought, since he let the students ask questions in and after class, while the teachers in Hong Kong would just go through examples on the chalk board and not care whether the students understood or not. He said that the Hong Kong system was pedantic and did not care for the students.

TEST SCORES

Arithmetic Reasoning: 6/10

Piagetian: 53/72

General Ability:
Analogies 19/24
Computation 13/26
Classification 19/26

Course Grade:
1 semester/D
2nd semester/D+

WORD ASSOCIATION TEST

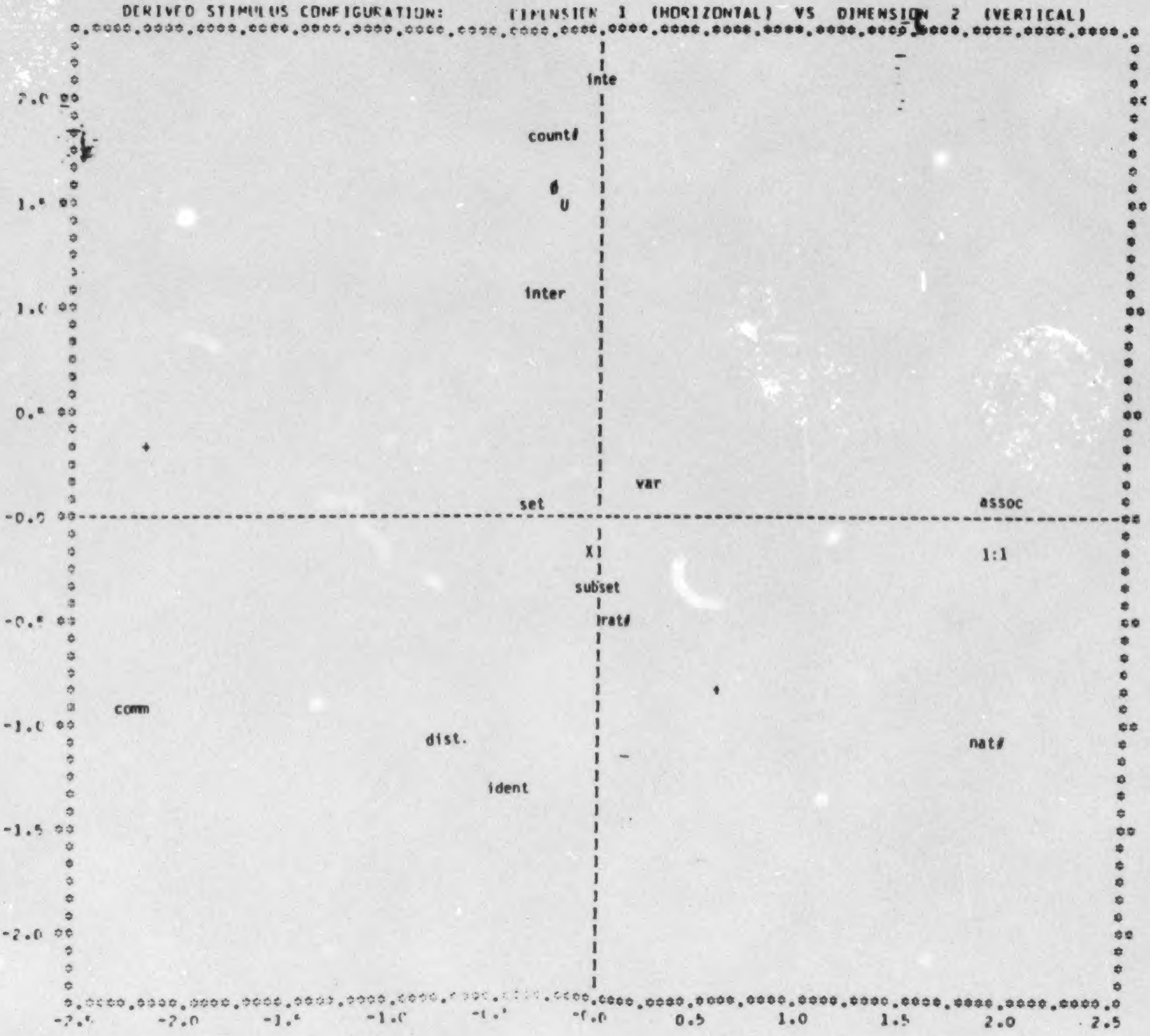
Despite Chung-Kwong's claim that he had forgotton most of the mathematics he had learned in Hong Kong and despite his late entry into the Algebra I class (in early November), he was one of the three target students who gave the correct type of responses to the Word Association Test. It is likely that his abandoning of the mathematics he had learned in Hong Kong actually enabled him to concentrate on what was taught during the first two months of the Algebra I class, which placed emphasis on the fundamental algebraic concepts. This absorption of the U.S. algebra curriculum without interference from that learned in Hong Kong probably enabled him to conceive the algebraic concepts within a related system, as was introduced in the textbook.

The following are the RC matrix derived from Chung-Kwong's responses and its corresponding two-dimensional graphical representation.

TABLE 2: RC matrix of Chung-Kwong Lee's Word Association Test responses.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1 Set	1.0																					
2 1:1		1.0																				
3 nat#		.411	1.0																			
4 integ				1.0																		
5 subset					1.0																	
6 comm	.232					1.0																
7 assoc	.224			.129		.043	1.0															
8 +								1.0														
9 ≠									1.0													
10 elem										1.0												
11 rat#											1.0											
12 U	.330			.281		.091	.167					1.0										
13 +			.060	.154									1.0									
14 count#	.400					.107	.171					.202		1.0								
15 dist	.470					.189		.202						.090	1.0							
16 –				.085												1.0						
17 ident	.282													.104			1.0					
18 expo																		1.0				
19 ∩	.436			.220		.072	.109					.218			.273		.179		1.0			
20 X																				1.0		
21 ∅	.188			.337		.106	.159					.400			.113				.283		1.0	
22 vari	.292																					1.0

FIGURE 2: Two-dimensional scaling solution of Chung-Kwong Lee's RC matrix.



SORTING TEST

Chung-Kwong sorted the twenty-eight concepts into six categories, the fewest used by the eight target students.

Group	Words/Concepts	Name of Group
(1)	x, $>$, $<$, U, +, +, −	Symbols
(2)	Quadratic, Equation, Polynomial	Factoring
(3)	Associative, Commutative, Factor, Identity, Distributive	Properties
(4)	Line, Point, Plane, Slope, Graph, Coordinate	Geometry
(5)	Absoluate Value, Element, Set, Exponent, Rational Number	Unclear
(6)	$\sqrt{}$, Radicals	

PROBLEM SOLVING

Chung-Kwong was not a high mathematics achiever. The Algebra I teacher, Mr. Wong, described him as very slow in math, but trying very hard and working diligently to follow the instruction. Chung-Kwong himself frequently said to the researcher that he was not very good in academic matters and that mathematics had been his worst subject. The results of his problem solving were not impressive. Of the forty-six problems administered to him, for only eight did he find the correct answers by himself. The researcher did guide him through several others. But for most of the problems, he just looked at them for a while and said that either he did not know how or had forgotten how to do them. Because of this, the researcher was not able to collect as much information on his problem-solving processes as on those of better problem solvers. Nevertheless, the following are his problem-solving features:

(1) Chung-Kwong tended to use methods taught in the Algebra I class to solve the problems, probably because he had left school for two years before coming to U.S. and had mentally abandoned the mathematics he had learned in Hong Kong.

Problem II-2: Factor $3x^2 + 7x + 2$.

Chung-Kwong: He used a procedure that was taught in the Algebra I class.

$$\frac{1}{3} \cdot 3[3x + 7x + 6]$$

$$\frac{1}{3}\left[(3x)^2 + 7(3x) + 6\right]$$

$$\frac{1}{3}\left[(3x+1)(3x+6)\right]$$

$$\frac{1}{3} \cdot 3 \; (3x+1)(x+6)$$

$$(3x+1)(x+6)$$

Problem II-10: Graph the solution set of $x + y = 1$.

Chung-Kwong: Following the method taught in the class, he wrote down a set of number pairs for the equation, plotted these number pairs on the graph, and drew a straight line through the points.

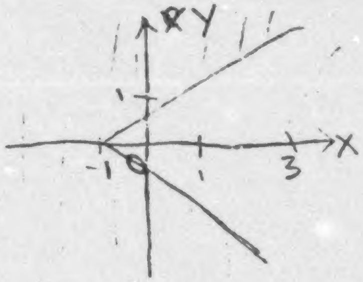
However, the following examples showed that he did not have a good understanding of what was taught in the class and often became confused by the many concepts and procedures.

Problem I-1: $8y^2 - 2y^2 - 2y^2 = ?$

Chung-Kwong: Instead of subtracting $2y^2$ from $8y^2$ and then subtracting $2y^2$ from $6y^2$, he changed $-2y^2-2y^2$ to $-(2y^2 + 2y^2)$, arrived at $4y^2$, and then subtracted $4y^2$ from $8y^2$ to obtain the correct answer.

Problem II-6: Graph $[x: |x + 1| < 3]$.

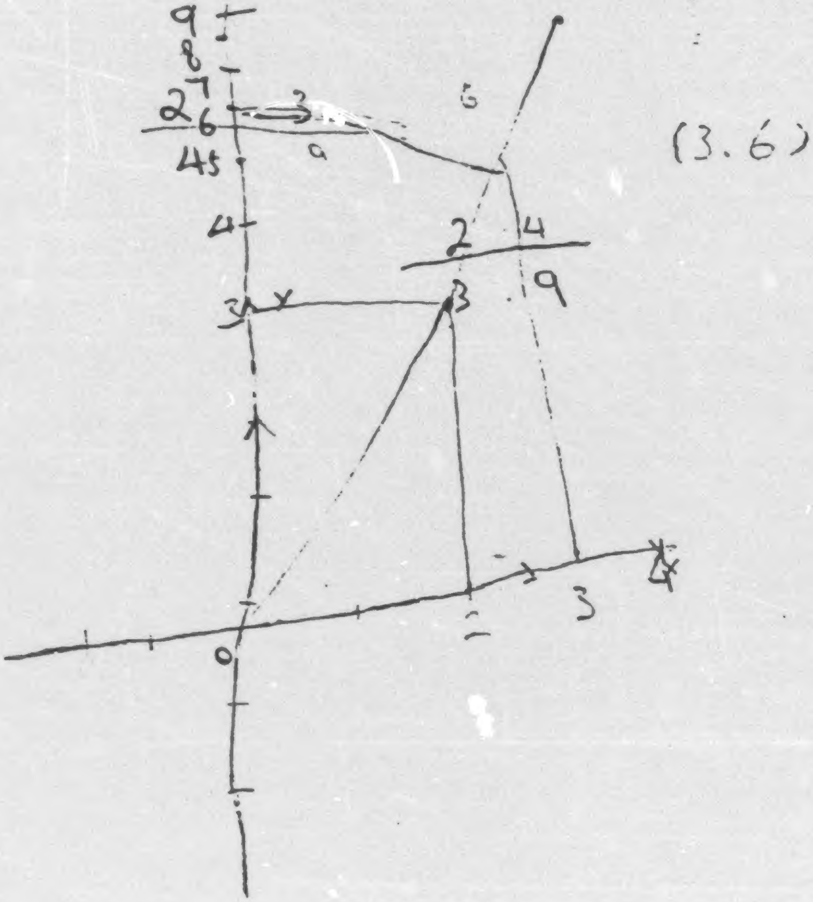
Chung-Kwong: He had associated graphs of absolute values with a V-shaped diagram, and for all such problems automatically drew a V-shape graph without any thinking. He arrived at the following solution to the above problem similarly.



Probably because of his lack of understanding of the concepts, he often forgot the formulae taught or he would remember a wrong one and not know how to verify it. Thus he was unable to do any of the problems on coordinate geometry except for the following one.

Problem II-14: Find the coordinates of the midpoint of the segment $(2,3), (4,9)$.

Chung-Kwong: For this problem, he did not try to remember the formula but went directly to the diagram and obtained the answer by inspection.



(2) The above example also illustrated another of Chung-Kwong's problem-solving characteristics. He was not dexterous in applying procedures or concepts to solve mathematics problems; if the problem was only somewhat different from the ones he had encountered in class, he would be stuck.

Problem II-7: If $(x + 7)^2 - 1 = 0$, x = ?

Chung-Kwong: He went at once to multiply out $(x + 7)^2$ and got $x^2 + 14x + 49 - 1 = 0$. But then he did not know how to factor the equation, and after looking at it for a while, he gave up. The researcher told him to subtract 1 from 49 and rewrite the equation as $x^2 + 14x + 48 = 0$. He was then able to complete the problem.

(3) When solving problems with absolute values, Chung-Kwong always replaced the absolute sign (l l) with the ± sign. The researcher asked him why he did that, since it was not taught in the class. He said that he learned it from another (target) student, Kung-Mon, who sat in front of him in class, and also that he remembered it from the mathematics lessons learned in Hong Kong. However, he could not do any of the problems with absolute value except for the easiest ones.

Problem I-8: Find the solution set: $|x| = 3$.

Chung-Kwong: He wrote automatically $x = +3$ and $x = -3$, and then $[x = +3, -3]$.

Probem I-10: Find the solution set $|x - 1| = 3$.

Chung-Kwong: He somehow equated the problem as $|x| - 1 = 3$ and arrived at the answer $[x = 4, -4]$.

(4) Chung-Kwong could only do one of the word problems. The researcher thought at first that he had difficulties with the English, but he actually has a very good command of English and was able to tell the researcher what each of the questions asked. The difficulty lay in his inability to transform the work problems into mathematical equations (sentences). The only problem he could do was the following:

Problem III-11: Find two integers whose sum is 18 and whose product is 72.

Chung-Kwong: He first wrote, "Let x is small integer" and "Let y is big integer." Then he wrote two equations, $x + y = 18$ and $xy=72$. Next, instead of solving the two simultaneous equations, he used the trial-and-error method and wrote the following:

1×17
3×15
2×16
4×14
5×13
6×12

The last product gave the desired 72 and therefore $x = 6$ and $y = 12$.

CONCLUSION

Despite his diligence and effort, Chung-Kwong did not learn the Algebra I contents very well. The school counselor was probably too ambitious to place him in the Algebra I course and so direct him into a technical profession which was not so dependent on English proficiency. Instead, he should have been placed in a general mathematics or pre-algebra course first and, depending on his performance, then placed into the college-prep mathematics series. The teacher, Mr. Wong, did not think that Chung-Kwong should pursue further the college-prep mathematics series, and gave him a D+ for the course, a grade which prevented him from doing so. It would be interesting to follow Chung-Kwong and learn what he chooses for his career alternatives; however, the scope of this study did not allow the researcher to pursue this further.

XI. CASE STUDY 6: CUONG KHON NGO

Born and brought up in Saigon, Vietnam, Cuong Khon left his home country with an older brother in 1979. After spending two weeks on a small boat with twenty people, they landed in Malaysia and spent seventeen months there in a refugee camp. After that, he and his brother were resettled in San Francisco, where they shared an apartment with two other young Vietnamese adults. His parents were still in Vietnam, because they could not afford the cost of the passage out. His father was a construction worker and his mother a housewife

Cuong Khon finished eighth grade and had three years of algebra in Vietnam. In Malaysia, he received English and U.S. cultural orientation training, but no instruction in other academic subjects. After arriving in San Francisco, he entered the Newcomer High School and took a year of general mathematics. This was his first year at Mission High School.

Mr. Wong said that Chuong Khon was very interested in mathematics and that he learned very fast and understood the concepts, although he was careless and made many mistakes.

PERCEPTION OF ALGEBRA I

Cuong Khon said that he was quite confused by the Algebra I contents in the first semesters, that were too many terms and concepts he had never heard of (e.g., real number, rational number, irrational number, etc.). Gradually, he did manage to grasp the ideas, but he said there were still concepts which he was not too sure about at the end of the school year.

He said that the class was a little slow here, that the ones in Vietnam were faster. He thought that the class here was easier because there was less homework, but that the mathematics here were very "troublesome."—he had to go through many more steps to get to an answer. He liked the methods in Vietnam better, because they made it easier and faster to get an answer. For example, in an equation, all he had to do was switch the positions of the numbers, change the signs, and he would have the answer. He admitted that the method taught in the Algebra I class was clearer.

TEST SCORES

Arithmetic Reasoning:	5/10
Piagetian:	51/72
General Ability:	Analogies 10/24 Computation 13/26 Classification 22/26
Course Grade:	1st semester/B- 2nd semester/B

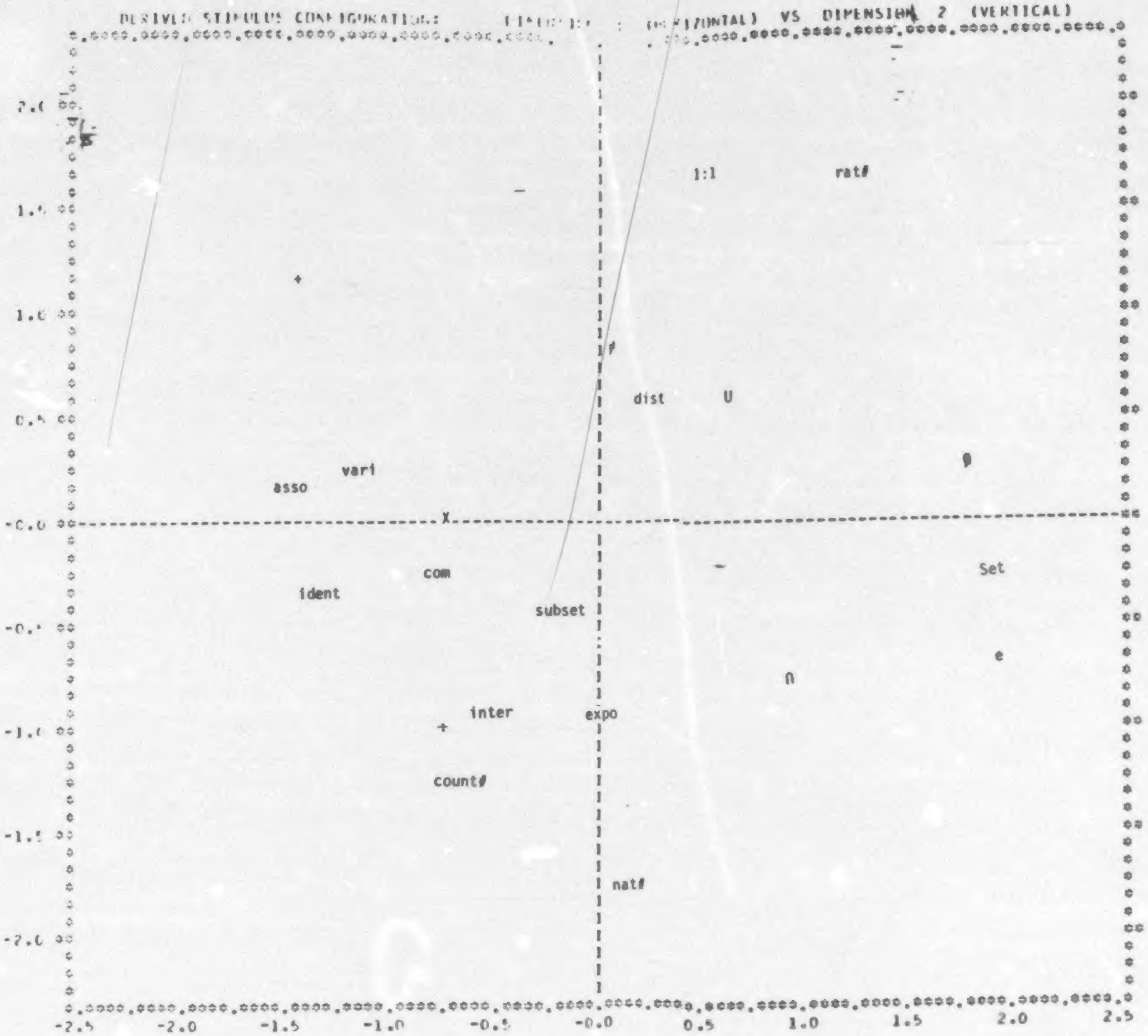
WORD ASSOCIATION TEST

While Cuong Khon wrote down the appropriate type of responses to most of the stimulus words/concepts, for three concepts ONE-TO-ONE CORRESPONDENCE, DISTRIBUTIVE PROPERTY, and UNION he wrote sentences explaining the terms, and for the word SET he wrote down some of its properties. The following are the RC matrix of Cuong Khon's responses and its corresponding two-dimensional scaling solution.

TABLE 3: RC matrix of Cuong Khon Ngo's Word Association Test responses.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1 Set	1.0																					
2 1:1		1.0																				
3 nat#			1.0																			
4 integ				1.0																		
5 subset	.217				1.0																	
6 comm			.147	.389		1.0																
7 assoc				.336		.230	1.0															
8 +							.304	1.0														
9 ≠									1.0													
10 elem	.182				.101					1.0												
11 rat#				.043		.074	.029				1.0											
12 U												1.0										
13 +				.091		.114	.470	.101			.023		1.0									
14 count#			.533			.355	.121				.303		.036	1.0								
15 dist															1.0							
16 −				.087		.132	.138				.174		.200	.055		1.0						
17 ident				.171		.148	.114				.029		.121	.121		.087	1.0					
18 expo				.239		.185	.145				.036		.036	.109		.055	.145	1.0				
19 ∩	.478			.129	.129	.124	.086			.439	.021		.091	.091		.065	.086	.109	1.0			
20 X				.214		.177	.471	.286			.043		.169	.169		.308	.171	.217	.129	1.0		
21 ∅	.421				.086					.386									.329		1.0	
22 vari				.562		.379	.340				.085		.178	.178		.162	.149	.186	.064	.197		1.0

FIGURE 3: Two-dimensional scaling solution of Cuong-Khon Ng's RC matrix.



SORTING TEST

Cuong Khon sorted the twenty-eight words/concepts into nine categories.

Group	Words/Concepts	Name of Group
(1)	Plane, ∩, Slope, Equation, Line, Coordinate, Graph	Geometry
(2)	+, −, x, +	Operations
(3)	Element, Set	Set theory
(4)	Commutative, Associative, Distributive	Properties
(5)	>, <	Inequality
(6)	√‾, Radical	Unclear
(7)	Factor, Polynomial	
(8)	Exponent, Quadratic	
(9)	Point, Identity, Rational Number, Absolute Value	

PROBLEM SOLVING

Cuong Khon's home language was Chinese (Cantonese). However, since he was brought up in Saigon and received all his education there, he was more fluent in Vietnamese, especially for academic contents. Therefore, his "think-aloud" problem-solving sessions were all in Vietnamese. The researcher, having no understanding of that language, depended on a translator, and so the analysis was partly based on an English translation. This indirect process is not as accurate, since many subtle changes of intonation or pace of speech were lost.

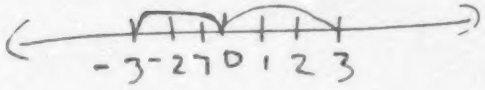
The following are Cuong-Khon's problem solving features:

(1) Despite his three years of lgebra in Vietnam, Cuong Khon used, almost exclusively, methods taught in the Algebra I class to solve the problems.

Problem I-8: Find the solution set: $|x| = 3$.

Cuong-Khon: Of the eight target students, he was the only one who used the number lines to solve this and other problems on absolute values. He never changed the absolute value sign to the $\pm$ sign, as all the other students did.

$|x| = 3 \qquad x = \{3, -3\}$



Problem I-12: Find the solution set -7x   4.

Cuong-Khon: He followed the procedure taught by the book. First he changed the inequality sign to the equal sign to obtain $-7x = 4$, from which he got $x = -4/7$. He located the point on the number line, selected two other points on the left and right of the point $-4/7$, and substituted the two points in the original inequality to determine on which side of the point $-4/7$ the solution set lay.

(2) Cuong Khon was also the only target student who routinely rechecked his answers.

Problem I-7: $(-4)^{37}(-1/4)^{37} = ?$

Cuong-Khon: He arrived at the correct answer by an incorrect process: $(-a)^b(-1/a)^b = (-a)(-1/a)$. He rechecked the answer by working a similar problem, $(-5)^3(-1/5)^3$.

Problem II-2: Factor: $3x^2 + 7x + 2$.

Cuong-Khon: He had no problem arriving at $(3x+1)(x+2)$, but he also multiplied the two factors again to check if he got the original polynomial.

(3) He did many of the problems by substituting actual numbers in the equations or inequalities, i.e., he was using induction.

Problem I-3: True or false? Why?
If $a > 0$, $b < 0$ and $|a| > |b|$, then $a+b = -(|a|+|b|)$.

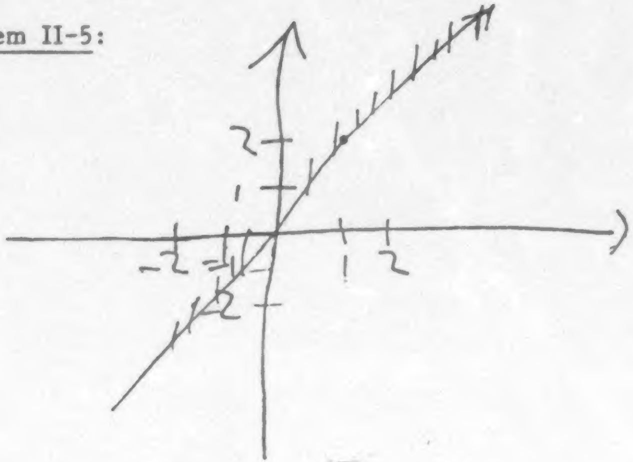
Cuong-Khon: He substituted 3 for a and -1 for b to prove that the statement was false.

Problem I-9: If $a > b$ and $k < 0$, then ka _?_ kb.

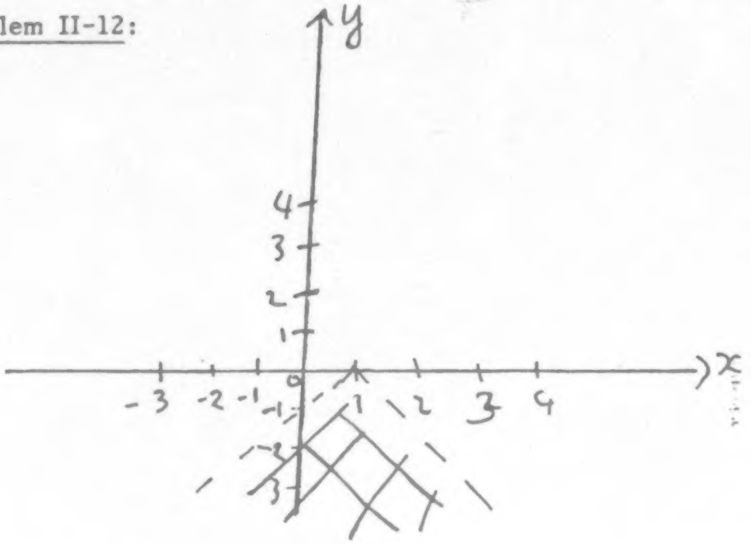
Cuong-Khon: He wrote down k=[-1,-2,-3], substituted 2 for a and 1 for b, and arrived at the correct answer.

(4) He could not do the problems on graphs except for the straightforward ones.

Problem II-5:



Problem II-12:



(5) He also had difficulties with word problems, though he was able to do the following one:

> Problem II-17: Two cars start from the same point and travel in opposite directions at the rate of 25 and 35 miles per hour, respectively. In how many hours will they be 330 miles apart?

But he was not able to do the following two problems, nor to set up the equations for them.

> Problem II-11: Find three consecutive integers such that the absolute value of their sum is twice the middle integer, increased by 28.

> Problem II-19: Mr. Lee's age is one year less than twice Miss Wong's age. The sum of their ages is 26. How old is each?

(6) He made many careless errors in his problem-solving processes.

Problem III-9:

$$x^{-2}y^{2}x^{-2}y^{-3} = x^{-4}y^{-1} = \frac{1}{x^{4}y^{1}}$$

CONCLUSION

All data indicated that Cuong-Khon had a very good understanding of the mathematics concepts taught in the Algebra I class. However, he was not as good at problem solving as he could be, since he often made careless mistakes in his computations and devivations. Therefore, he was not able to do the more complicated or lengthy problems.

XII. CASE STUDY: SANG-CAM SU

Sang-Cam Su is an Indochinese refugee who came to the U.S. in April 1981. Sang-Cam left his home town, a small Vietnamese village near the Chinese border, in 1979 with his parents and five younger brothers. After spending two months on a 10-ton boat with forty other people, he arrived in Hong Kong, where he lived in a refugee camp for one year. After that, his family was transferred to another refugee camp in the Philippines, where he spent another six months before coming to the United States.

Sang-Cam graduated from a Chinese elementary school in Vietnam and was attending a Vietnamese middle school when he left his hometown. He had one year of algebra in middle school. Though he received some English and cultural orientation training while at the refugee camps in Hong Kong and Philippines, he did not receive any formal instruction in mathematics there.

Sang-Cam's parents were born in China and immigrated to North Vietnam. His father was a construction worker and his mother a housewife. After arriving in the U.S., the father has been attended adult school and the mother stayed at home.

In class, Sang-Cam was attentive. The teacher described him as naive, quiet, nonaggressive, and hard working.

PERCEPTION OF ALGEBRA I

Sang-Cam thought that the Algebra I course was very difficult. He found the teacher, Mr. Wong, very good and his explanations very clear, but he still could not understand the concepts. And once he did not understand, he was lost completely. He liked the teaching method in Vietnam better, where the formulae were taught as songs. They were easy to remember and when he encountered difficulty, all he had to do was remember the song. Here, there were few formulae. The few taught in class were in English; they did not rhyme and were hard to remember. He also complained that Mr. Wong did not correct their homework. In Vietnam, the teacher would check their homework individually and asked the the students to redo what was incorrect. Thus he knew his mistakes and what concepts to relearn.

TEST SCORES

Arithmetic Reasoning:	5/10
Piagetian:	50/72
General Ability:	Analogies 12/24 Computation 12/26 Classification 21/26
Course Grade:	1st semester/C- 2nd semester/C-

WORD ASSOCIATION TEST

Sang-Cam wrote his responses half in English and half in Vietnamese. Most were examples or explanations of the stimulus words/concepts. Because of the inappropriateness of his responses, no further analysis was performed on his word association data. The following were the four exceptions, when he wrote the appropriate type of responses:

NATURAL NUMBER: whole number, counting number, real number

INTEGER: whole number, even number

÷ : subtract, division, multiplication, give one to each

ELEMENT: set, become, parallel, equality

SORTING TEST

Sang-Cam sorted the twenty-eight concepts into six groups:

Group	Words/Conepts	Name of Group
(1)	Element, Plane, Set, Line	Geometry
(2)	Associative, +, ÷, −, X	Operations
(3)	$<$, $>$	Inequality
(4)	U, Distributive, Commutative, Point, Graph	Unclear
(5)	Slope, Quadratic, Factor, Coordinate, $\sqrt{\ }$, Radical	
(6)	Absolute Value, Polynomial, Identity, Equation, Exponent, Rational Number	

PROBLEM SOLVING

Sang-Cam was not an exceptional mathematics achiever in the class. The teacher considered him below average but thought he tried very hard and worked diligently to learn the subject. The following are his problem-solving features:

(1) Sang-Cam depended to a great extent on formulae. For example, when he could not do a problem, he would say that he could not remember the formula. The following is an example of Sang-Cam applying an incorrect formula to a problem:

Problem I-7: $(-4)^{37}(-1/4)^{37} = ?$

Sang-Cam: $= (-4)(-1/4)^{37+37} = (1)^{74} = 1 \times 74 = 74.$

(2) When dealing with problems on absolute values, he used the ± substitution exclusively and not the number line as taught in class. Even if the problem asked him to graph the solution set, he would still find the numerical solution first before graphing the solution onto the number line.

Problem I-11: Graph $|x| < 2$.

Sang-Cam: He saw at once that the answer is the set between -2 and 2. He then drew the number line and graphed the solution.

(3) Sang-Cam, like Rose Huyen, liked to carry out the computations or operations in his mind instead of writing down the steps. However, different from Rose, he was more accurate and seldom made errors.

Problem III-5: Simplify $\frac{2}{a-2} + \frac{1}{a^2-4} + \frac{3}{a+2}$

$$= \frac{2a+4}{a^2-4} + \frac{1}{a^2-4} + \frac{3a-6}{a^2-4} = \frac{5a-1}{a^2-4}$$

Problem III-9: $\frac{x^{-2}y^2}{x^2y^{-3}} = ?$

$$= \frac{y^5}{x^4}$$

(4) Sang-Cam was always deterred by the English in the problems. Sometimes, even when he understood every English word, he still said that he did not know what the problem was asking for. The researcher, on several problems, had to explain the problems to him repeatedly in Chinese before he would know what to do. It was not clear whether it was the English that he did not understand or the mathematics problem itself. The following are examples of problems that he had difficulties with.

Problem I-3: True or false? Why?

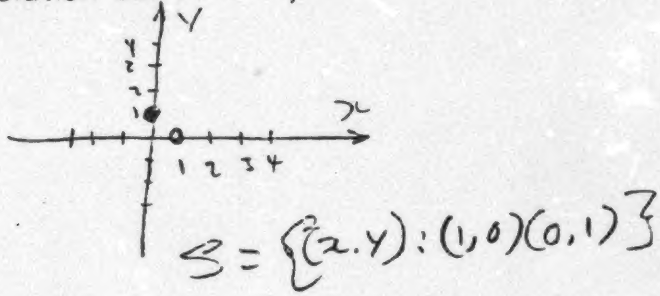
If $a > 0$, $b < 0$ and $|a| > |b|$, then $a+b = -(|a| - |b|)$.

Problem I-4: Under what condition does $|a+b| = -(a+b)$?

Problem I-5: If $x \geq 0$ and $y \geq 0$, the smallest value of $2x+y^2-1$ is ?

(5) Sang-Cam could not do the graph problems except for those that could be plotted by number pairs. He did not seem to have a grasp of the relationship between two-dimensional space and quadratic equations.

Problem II-10:
Graph the solution set of $x + y = 1$.



Problem III-10: Graph $y \geq (x-2)^2 + 3$.

Sang-Cam: He multiplied out $(x-2)^2 + 3$ to get x^2-4x+7. Then he calculated seven number pairs for $y = x^2-4x+7$, plotted these points on the graph, and shaded in the solution set.

CONCLUSION

Sang-Cam did not learn the contents of the Algebra I class very well. He did not have a good understanding of the concepts, probably because he picked up all the expedient formulae from classmates to do the assignments. A novice in using the formulae, he was not very dexterous in applying them in problem solving. Maybe his placement in the Algebra I class was premature and he should instead have been enrolled in a pre-algebra course to learn the basic mathematics concepts.

XIII. CASE STUDY 8: DOUGLAS YEUNG

Douglas Yeung left Guangzhou (Canton), China in 1978 with his parents and a younger brother. They went to Hong Kong first, stayed there for three years, then moved to San Francisco in August 1981. Both of his parents had been teachers in China, his father at a middle school and his mother at a pre-school. After coming to the U.S., his father got a job with the San Francisco Unified School District as a janitor and his mother attended adult school.

Douglas was attending seventh grade at a "main point" middle school in Guangzhou when his family moved to Hong Kong. These were schools which received special allocation for teachers and resources and were considered to be the best in China. Admittance was by competitive examination. In Hong Kong, he repeated seventh grade at an English middle school and continued through the nineth grade. He had taken algebra since the seventh grade in China.

PERCEPTION OF ALGEBRA I

Douglas considered the Algebra I course to be very easy and did not study very hard. He was more concerned about the English terminologies and said that he often checked the dictionary and referred to the text to check the spellings.

He thought that mathematics courses were mainly dependent on the textbooks and that the basic contents taught in China, Hong Kong, and the U.S were similar. The main differences were in emphasis and sequencing of the topics. In China, his algebra teacher asked students to copy and recite definitions day after day and there was more writing than problem solving in the class. In Hong Kong, there was testing every day, and the emphasis was on the scores. You had to get the correct answer and they did not care how you got it. In the U.S., the teacher emphasized the basic concepts; he considered this approach somewhere between those used in China and Hong Kong. He probably thought that remembering definitions was the same as understanding concepts.

TEST SCORES

Arithmetic Reasoning:	10/10
Piagetian:	49/72
General Ability:	Analogies 20/24 Computation 20/26 Classification 20/26
Course Grade:	1st semester/A 2nd semester/A

WORD ASSOCIATION TEST

Douglas wrote down his responses entirely in Chinese. He gave down definitions of the stimulus words/concepts or paragraphs explaining them. When interviewed, he said that his mathematics teacher in China, who was highly respected in the school, emphasized the memorization of definitions. Student had to recite them everyday. The teacher told them that if they could memorize the definitions, they would understand them.

Because of the inappropriate type of responses given by Douglas, no further analysis was performed on the word association data.

SORTING TEST

Of the twenty-eight words/concepts, Douglas sorted twenty-seven of them into eight groups. One word/concept, IDENTITY, was left by itself. Dounglas said that he had forgotten what an identity was and was unable to relate it to other words/concepts. The following are the eight categories:

Group	Words/Concepts	Name of Group
(1)	U, $<$, $>$, +, x, $\sqrt{\ }$, -, ÷	Symbols
(2)	Commutative, Associative, distributive	Properties
(3)	Graph, Coordinate, Equation, Slope	Graph
(4)	Line, Plane, Point	Geometry
(5)	Radical, Exponent, Quadratic	Quadratic forms
(6)	Polynomial, Rational Number	Unclear
(7)	Absolute Value, Set, Element, Factor	

PROBLEM SOLVING FEATURES

Douglas acquired a reputation in the Algebra I class as a high mathematics achiever. He had quite a group of followers who often asked him to help them with their problems. In the interview, he said that the Algebra I class was too easy. Classroom observation showed that he often ignored the instruction and involved himself in work from other classes. He said that he would look at the examples in the textbook when he could not do his homework. Probably because of his laissez-faire attitude towards the Algebra I class, he did not perform as well as his reputation indicated in the problem solving sessions. The following are his problem-solving features:

(1) Douglas was very good in using rules and formulae to do his problems, and could usually solve them expediently and accurately.

However, he depended on the rules and formulae so much that he seldom tried other methods.

Problem II-14: Find the coordinates of the midpoint of the segment $(2,3), (4,9)$.

Douglas: He looked at the problem for a while, said that he did not remember the exact formula, and went on to the other problems. Towards the end of the session, Douglas went back to it. But he still said that he could not remember the formula. The researcher asked him to use a diagram to do the problem. He followed the suggestion and arrived at the correct answer, but still said that he was not sure about the answer because he did not remember the formula.

Problem I-9: If $a > b$ and $k < 0$, then $ka \underline{\quad ? \quad} kb$.

Douglas: At first, he tried to substitute actual numbers into the inequality. Suddenly, he remembered the rule that when both sides of an inequality are multiplied by a negative number, the inequality sign has to be reversed. He wrote down the correct answer.

(2) While Douglas was very good in doing problems that ask for specific answers, he was not able to solve most of those asking for explanations or proofs. For example, he could not do the following problems:

Problem I-3: True or false? Why?
If $a > 0$, $b < 0$ and $|a| > |b|$, then $a+b=-(|a|-|b|)$.

Problem I-5: Under what conditions does $|a+b| = -(a+b)$?

Problem II-13: The operation * is defined as : $a*b = 2a + b$.
Is * commutative? Why?
Is * associative? Why?

(3) Douglas would check his answers when he did not feel confident about them. But often, if the operations were smooth, he would not recheck his answer.

Problem II-4: If $k^3 + 2k^2 - 24k = 0$, then $k = ?$

Douglas: He arrived easily at the three answers (0, -6, 4) and began substituting them back into the orginal equation to check the answers. In the process, he made some careless mistakes, and so rejected -6 and 4 as answers and accepted 0 as the only answer. He might also have used this checking process because he was not sure about arriving at three roots.

Problem III-6: $x = \sqrt{x-2} + 4$, $x = ?$

Douglas: He went through the process and arrived at two answers (3, 6). But he did not border to recheck if both are real roots of the equation, probably because he went through the steps so smoothly that he did not think there was a possibility of error.

(4) Douglas could not do any of the problems asking him to graph inequality of absolute values. In the following two problems, he drew straight lines instead of aV-shape graph to represent absolute values.

Problem II-12: Graph $[(x,y) : y < |x+1|]$.

Problem II-16: Graph $[(x,y): |x-1| \leqslant 1] \cap [(x,y): y \leqslant |x|]$.

CONCLUSION

Douglas had learned the mathematics contents taught in China very well and was dexterous in solving problems which required the application of rules and formulae, as had been emphasized in China and Hong Kong. His ability in solving this type of problem probably gave the other students an impression of a excellent mathematics achiever.

He himself probably also thought that the materials covered in the course were too easy since he had learned most of the contents before (though the emphasis was not the same). He did not put much effort into the Algebra I course, and consequently both his performance in the problem-solving sessions and his course grade were disappointing.

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APPENDIX A

Arithmetic Reasoning Test

This test consists of 10 problems in arithmetic. However, you do not have to find the to each problem. You only have to tell HOW the answer could be found. Although some s may be worked more than one way, only one of the correct ways will be given among the choices.

Example 0. Mary's father was 26 years old when Mary was born. Mary is now 8 years old. How old is her father now?

(A) subtract
(B) divide
(C) add
(D) multiply

0. A B [✓ C] D

The age of Mary's father is found by adding 26 and 8, so choice (C) should be circled.

Example 00. Desks are priced at $40 each. If they are bought in lots of 4, the total price is reduced by $20. How much would 4 desks cost?

(A) divide, then add
(B) multiply, then multiply again
(C) subtract, then divide
(D) multiply, then subtract

00. A B C [✓ D]

One way to solve the problem would be to multiply $40 by 4 and then subtract $20 from the product. Choice (D) should be circled. DO NOT GUESS WILDLY. You should guess only if you can rule out some of the choices. ill have 5 minutes for this section. If you finish before time is called, check your work.

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

算數推論測驗

這項測驗包括10個算術題.你不需每題寫出答案.你只要說明如何找出答案的方法.

雖然有些問題的答案不只一種方法可求出.答案中只有一個是對的.

一.例題:瑪利的父親26歲時生瑪利,瑪利現在8歲.他父親現在幾歲?

(A)減 (B)除 (C)加 (D)乘

答案. A. B. [✓ C] D

瑪利父親的歲數是26加8求出的.所以選(C)

二.例題、書桌每張$40.如果一次購買4張.則可便宜$20,問4張桌子需多少錢?

(A)先除再加 (B)先乘再乘 (C)先減再除 (D)先乘再減

答案 A B C [✓ D]

有一方法解這問題是將$40乘4.然後由乘積中減$20,所以答案是(D).

不要亂猜答案,除非你能確定其他答案是錯的.你才可猜.

你有5分鐘時間做這部份的問題.如果你在期限內做完.請檢查你的答案.

不經考場人員指示不要翻到下一頁

1. John wants to buy a football costing $10.25. He has saved $8.50. How much more money does he need to save?
約翰想買一個$10.25的足球，他已儲蓄了$8.50，問他還需儲蓄多少錢？

   (A) subtract 減
   (B) add 加
   (C) divide 除
   (D) multiply 乘

2. Five boys share three melons equally. What is each boy's share?
5個男孩平均來分三個甜瓜，每個男孩能分多少？

   (A) add 加
   (B) divide 除
   (C) subtract 減
   (D) multiply 乘

3. A ham weighing $16\frac{3}{4}$ pounds was cut into two pieces. If one piece weighed $9\frac{1}{3}$ pounds, what was the weight of the other piece?
重$16\frac{3}{4}$磅的火腿切成兩塊，如果一塊的重量是$9\frac{1}{3}$磅，問另一塊重量多少？

   (A) subtract 減
   (B) divide 除
   (C) multiply 乘
   (D) add 加

4. A man wants to seed a lawn around his new home. His lot is 120 feet by 90 feet (10,800 square feet). His house is centered on the lot and occupies 2,785 square feet. How many square feet of ground may be put into lawn?
某人想在他新屋周圍種一片草地，他擁有地的面積是120呎乘90呎(10,800平方呎)。他的房子在地中央佔地2,785平方呎。問還有多少平方呎的地能被用來當草地？

   (A) add 加
   (B) divide 除
   (C) multiply 乘
   (D) subtract 減

5. There are 4 quarts in a gallon and 4 cups in a quart. What is the number of cups in a gallon?
一加侖有4夸脫，一夸脫有4杯。問一加侖有多少杯？

   (A) add 加
   (B) subtract 減
   (C) multiply 乘
   (D) divide 除

GO ON TO THE NEXT PAGE.

請接下頁

6. A wholesale meat dealer sells sirloin steak for $2.72 per pound and chuck steak for $1.91 per pound. One day he sold 79 pounds of each. How much money was taken in?

某批發肉商賣牛腰肉每磅$2.72，牛頭肉每磅$1.91. 某天兩種肉各賣79磅，問他共得多少錢？

(A) add, then divide 先加再除
(B) add, then multiply 先加再乘
(G) multiply, then subtract 先乘再減
(D) divide, then divide again 先除再除

7. A cyclist in an international bicycle race has covered an average of 9 miles every 20 minutes. If he can maintain the same average speed, how long will it take him to cycle the remaining 84 miles of the race?

某單車選手在國際單車賽中平均20分鐘走9哩，如果他能維持這個平均速度，問還需多少時間他能完成餘程84哩？

(A) divide, then multiply 先除再乘
(B) subtract, then divide 先減再除
(C) add, then subtract 先加再減
(D) divide, then add 先除再加

8. A store sells pencils for 96 cents a dozen. The pencils cost the store 75 cents a dozen. How much profit is there on each pencil?

某商店鉛筆每打賣96仙，每打鉛筆成本75仙，問每枝鉛筆的利潤是多少？

(A) subtract, then multiply 先減再乘
(B) divide, then subtract 先除再減
(C) add, then divide 先加再除
(D) subtract, then divide 先減再除

GO ON TO THE NEXT PAGE.

請接下頁

9. A certain cut of beef costs $1.50 per pound. How much beef could a housewife serve to each of 5 people if she could only afford to spend $4.00 for the beef?

牛肉每磅$1.50，某主婦只有$4.0買牛肉，問如果分給5個人每人得多少牛肉？

(A) divide, then divide again 先除再除

(B) multiply, then add 先乘再加

(C) subtract, then multiply 先減再乘

(D) divide, then multiply 先除再乘

10. At the beginning of a month, a car rental organization rented 37 cars. During the month, 32 of these cars were returned. If, at the end of the month, 43 of their cars were being rented, how many new rentals had been made?

月初租車公司租出車子37輛，月中回來了32輛．若到月底時共租出43輛車子，問月中以後新租出的車是多少輛？

(A) subtract, then divide 先減再除

(B) subtract, then subtract again 先減再減

(C) multiply, then subtract 先乘再減

(D) multiply, then add 先乘再加

END OF THE TEST.

試題完畢

APPENDIX B

Word Association Test: Stimulus Words/Concepts

WORD ASSOCIATION TEST
Stimulus Words/Concepts

SET 集
ONE TO ONE CORRESPONDENCE 一一相應
NATURAL NUMBER 自然數
INTEGER 整數
SUBSET 子集
COMMUTATIVE PROPERTY 交換律
ASSOCIATIVE PROPERTY 結合律
+
INEQUALITY 不等
ELEMENT 元
RATIONAL NUMBER 有理數
UNION U 并
+
COUNTING NUMBER 數數
DISTRIBUTIVE PROPERTY 分配律
–
IDENTITY 恒等
EXPONENT 指數
INTERSECTION 相交
X
∅
VARIABLE 變數

APPENDIX C

Phase I Algebra Test Items

1. $8y^2 - 2y^2 - 2y^2 = ?$

2. $13m - m + 3b - 2b = ?$

3. True or false? Why?
If $a > 0$, $b < 0$ and $|a| > |b|$,
Then $a + b = -(|a| - |b|)$

4. Under what conditions does $|a + b| = -(a + b)$?

5. If $x \geq 0$ and $y \geq 0$, the smallest value of $2x + y^2 - 1$ is ___________.

6. If $a - b = 3$, what is the value of $b - a$?

7. $(-4)^{37}(-\frac{1}{4})^{37} = ?$

8. Find the solution set: $|x| = 3$

9. If $a > b$ and $k < 0$, then ka___?___kb.

10. Find the solution set $|x - 1| = 3$

11. Graph $|x| < 2$

12. Find the solution set $-7x \leq 4$

APPENDIX D

Phase II Algebra Test Items

1. If $3x + 2x - 1 = x + 3$, $x = ?$

2. Factor: $3x^2 + 7x + 2$

3. What is the slope of the line: $3x - 5y = 15$?

4. If $k^3 + 2k^2 - 24k = 0$, then $k = ?$

5. Graph all points (x, y) such that x and y are both between 1 and 2.

6. Graph $[x : |x + 1| < 3]$

7. If $(x + 7)^2 - 1 = 0$, $x = ?$

8. $U = [1, 2, 3]$, graph $[(x, y): y > 2/x]$

9. The statement, "A certain number, f, increased by twice another number, n, is equal to 30," can be written as:

10. Graph the solution set of $x + y = 1$.

11. Find three consecutive integers such that the absolute value of their sum is twice the middle integer, increased by 28.

12. Graph: $[(x, y) : y < |x + 1|]$.

13. The operation * is defined as: $a * b = 2a + b$
    Is * commutative? Why?
    Is * associative? Why?

14. Find the coordinates of the midpoint of the segment $(2,3)$, $(4,9)$.

15. $6987 \times 7013 = ?$

16. Graph: $[(x, y): |x - 1| \leqslant 1] \cap [(x, y): y \leqslant |x|]$

17. Two cars start from the same point and travel in opposite directions at the rate of 25 and 35 miles per hour, respectively. In how many hours will they be 330 miles apart?

18. Is the following proof correct? Why?
    Let $x = 2$ and $y = 1$. Then,

$$\begin{aligned} x^2 - 2xy + y^2 &= x^2 - 2xy + y^2 \\ x^2 - 2xy + y^2 &= y^2 - 2xy + x^2 \\ (x - y)^2 &= (y - x)^2 \\ (x - y) &= (y - x) \\ 2x &= 2y \\ x &= y \\ 2 &= 1 \end{aligned}$$

19. Mr. Lee's age is one year less than twice Miss Wong's age. The sum of their ages is 26. How old is each?

20. Factor: $84x^2 + 135xy - 264y^2$

APPENDIX E

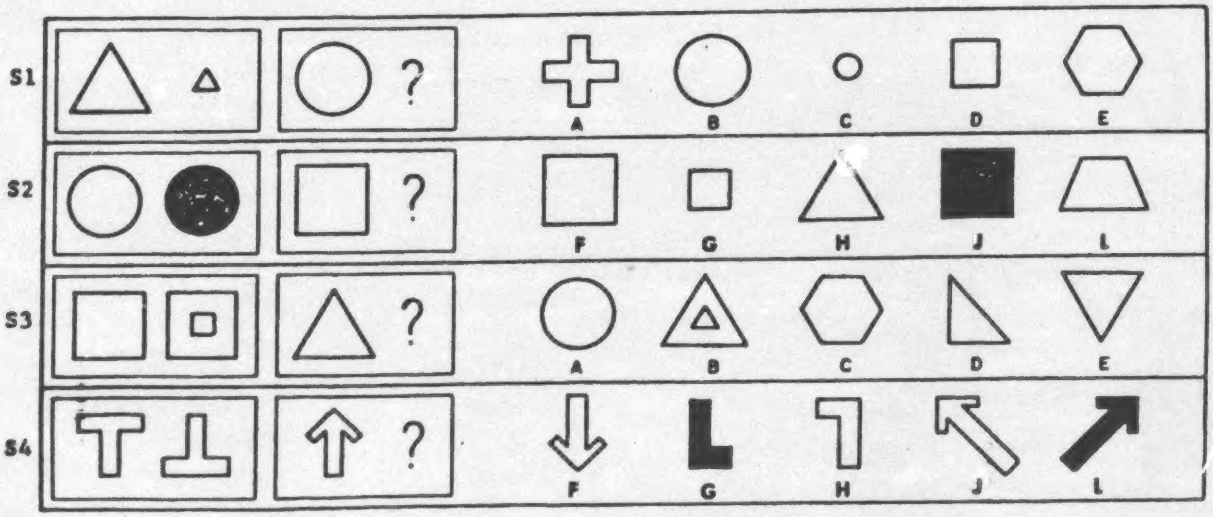
Cognitive Test

Phase III Data
6/15/82

NAME: ______________________________

DIRECTIONS, PART II, ANALOGIES

In each row find the drawing which will make the second pair of drawings like the first pair. Then look at the answer sheet, Part II, to see how the answer is marked.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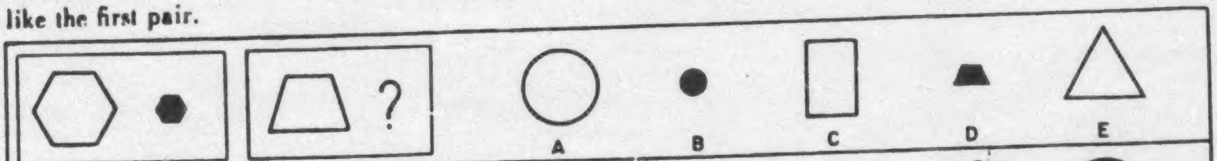
Do not turn the page until you are told to do so.

PART II. ANALOGIES

In each row find the drawing which will make the second pair of drawings like the first pair.

1



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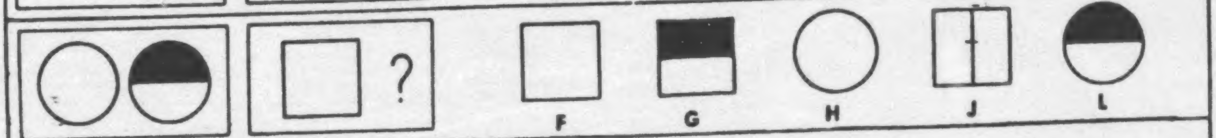

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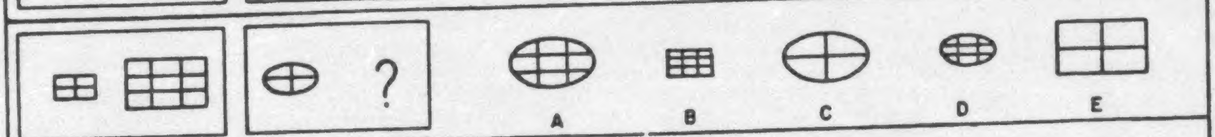

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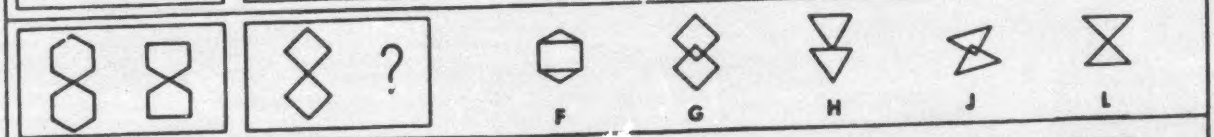

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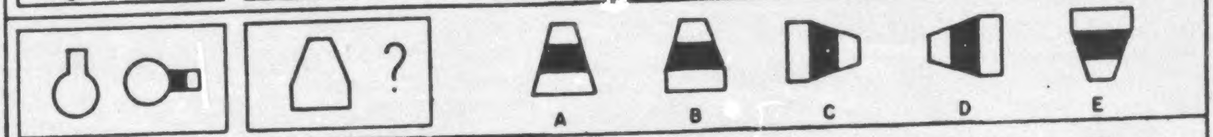

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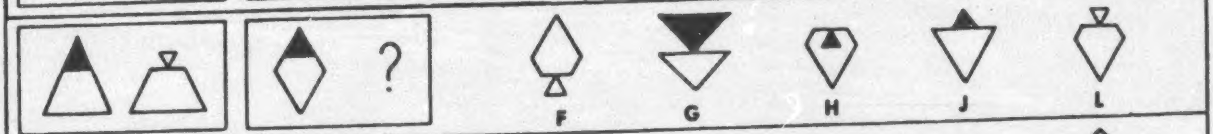

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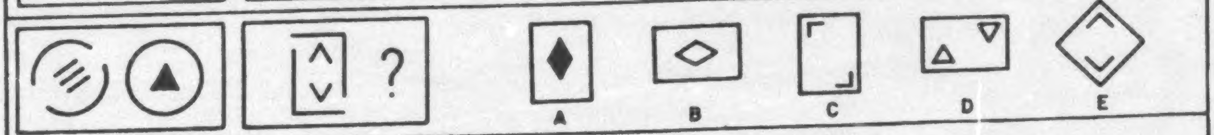

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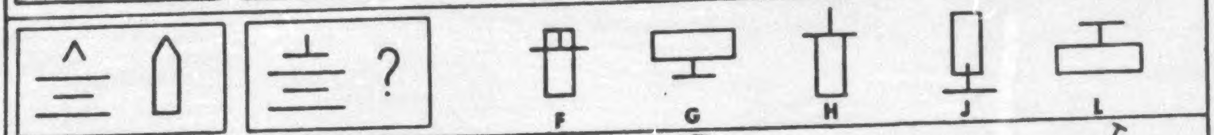

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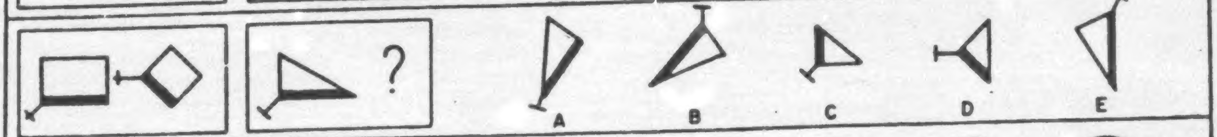

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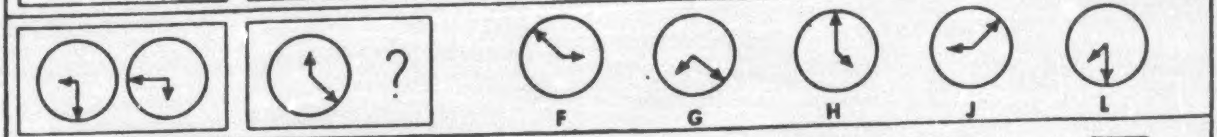

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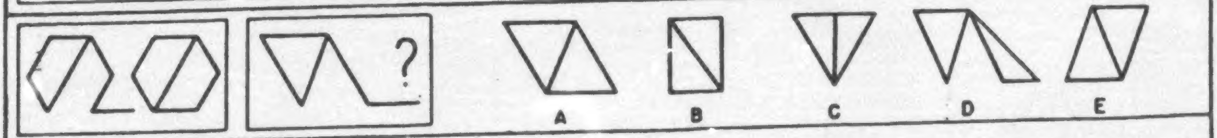

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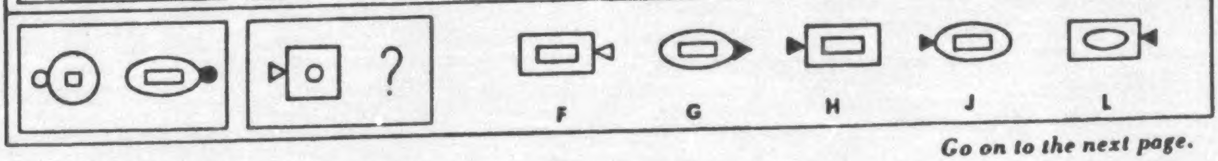

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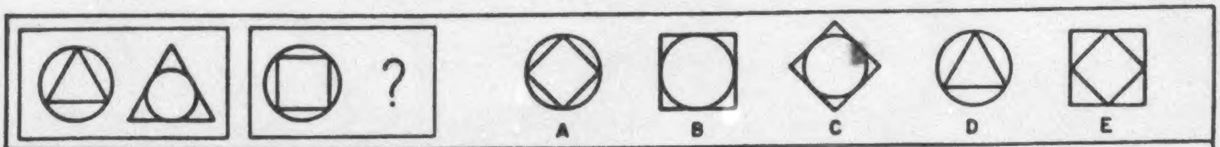

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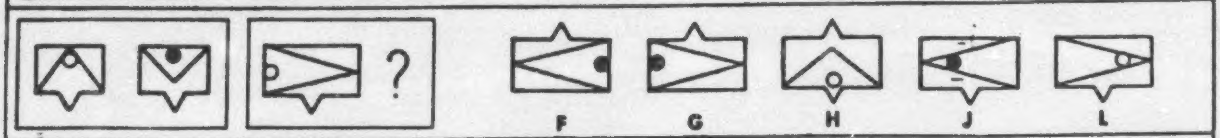

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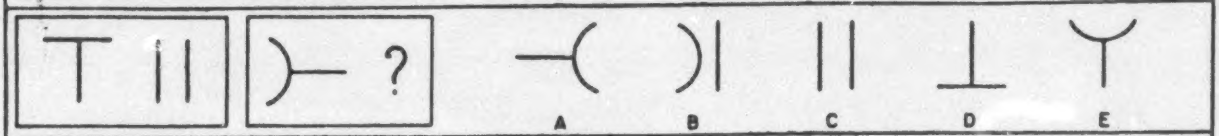

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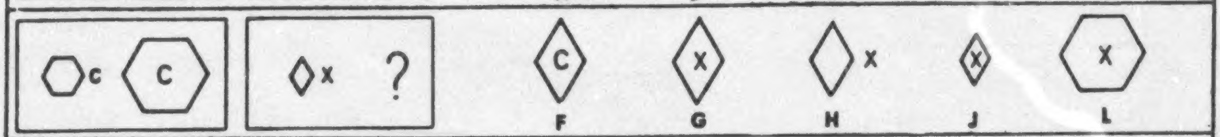
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C
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X
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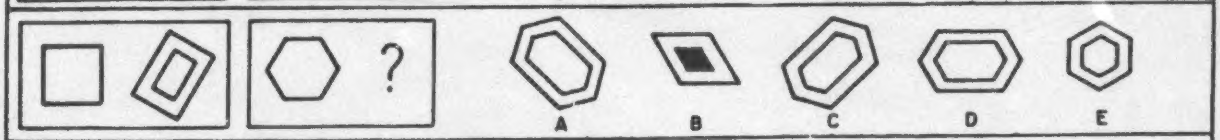
L

17


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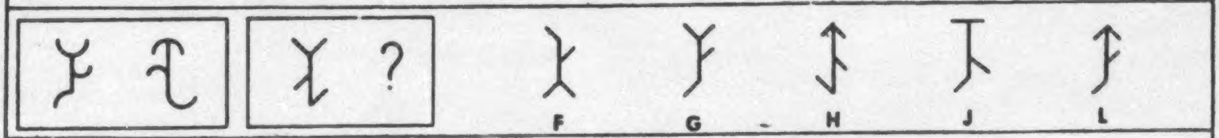

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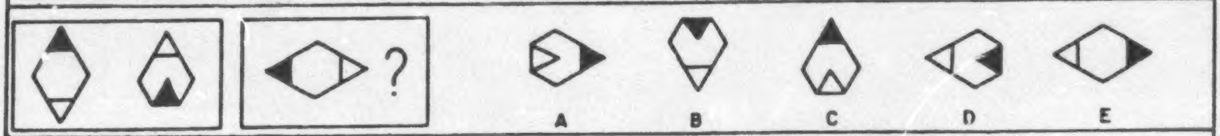

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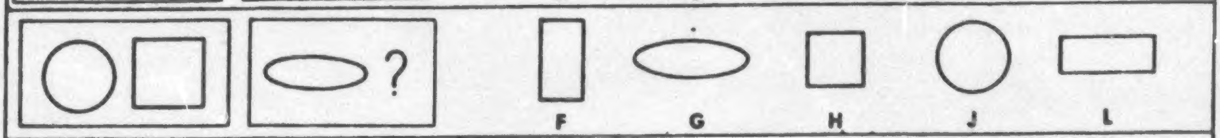

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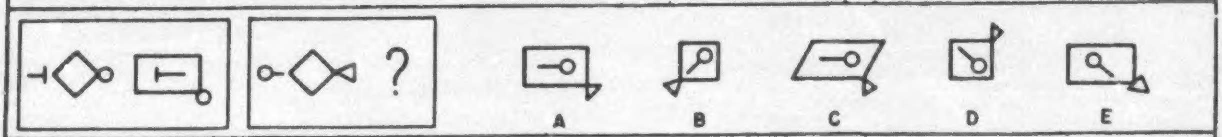

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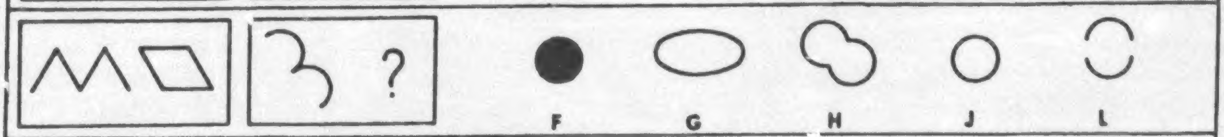

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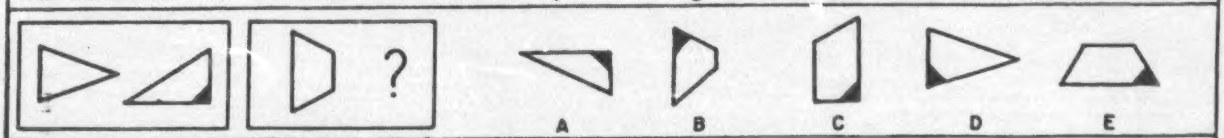

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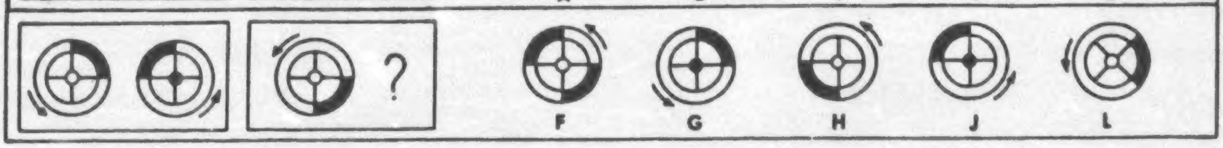

STOP. Wait here for further directions.

DIRECTIONS, PART III, COMPUTATION

In each problem find the right answer. Then look at the answer sheet, Part III, to see how the answer is marked.

T1 $\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$

A 8
B 10
C 9
D 11
E None of these
(Answer B is correct.)

T2 $\begin{array}{r} 10 \\ \times 3 \\ \hline \end{array}$

F 22
G 25
H 20
J 33
L None of these

T3 $20 \div 5 = ?$

A 3
B 5
C 6
D 4
E None of these

Wait here for further directions.

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PART III, COMPUTATION

In each problem find the right answer.

1 $$\begin{array}{r} 63 \\ -\ 26 \\ \hline \end{array}$$

F 36
G 43
H 37
J 47
L None of these

2 $$\begin{array}{r} 38 \\ +\ 49 \\ \hline \end{array}$$

A None of these
B 76
C 77
D 86
E 88

3 $$\begin{array}{r} 947 \\ \times\ 8 \\ \hline \end{array}$$

F None of these
G 7576
H 7276
J 7565
L 7525

4 $2583 \div 7 = ?$

A $368\frac{4}{7}$ B $366\frac{1}{7}$
C 373 D 369
E None of these

5 $120 \times 3\frac{3}{4} = ?$

F $360\frac{3}{4}$ G 430
H None of these
J $440\frac{1}{4}$ L 450

6 $6\frac{1}{3} + 5\frac{1}{6} = ?$

A $11\frac{5}{12}$ B $11\frac{1}{2}$
C $11\frac{2}{9}$ D $11\frac{2}{3}$
E None of these

7 $\frac{4}{5} = \frac{52}{?}$

F 80
G 60
H 75
J 85
L None of these

8 $3\frac{2}{5} \div 2 = ?$

A $1\frac{7}{10}$ B $2\frac{3}{5}$
C $1\frac{2}{5}$ D $1\frac{3}{7}$
E None of these

9 $\frac{4}{3} \times 2\frac{1}{2} = ?$

F $2\frac{2}{3}$ G $1\frac{7}{8}$
H $3\frac{1}{2}$ J $3\frac{1}{3}$
L None of these

10 $\frac{5}{7} - \frac{4}{21} = ?$

A $\frac{9}{7}$ B $\frac{1}{14}$
C $\frac{11}{21}$ D $\frac{1}{7}$
E None of these

11 $2\frac{1}{3} \times 8 = ?$

F $16\frac{1}{3}$ G $17\frac{2}{3}$
H $19\frac{1}{3}$ J $18\frac{2}{3}$
L None of these

12 $\frac{3}{4} \div \frac{2}{3} = ?$

A $1\frac{1}{4}$ B $1\frac{1}{8}$
C $\frac{5}{12}$ D $1\frac{1}{3}$
E None of these

13 $4 \div 1\frac{1}{3} = ?$

F $3\frac{2}{3}$ G $4\frac{1}{3}$
H $\frac{1}{3}$
J None of these
L 4

14 $\frac{1}{2} + \frac{7}{10} = ?$

A $\frac{9}{10}$ B $1\frac{3}{10}$
C $1\frac{1}{5}$ D $\frac{8}{12}$
E None of these

15 $\frac{2}{3} \div 1\frac{1}{9} = ?$

F $1\frac{2}{27}$ G $\frac{2}{9}$
H $2\frac{1}{27}$
J None of these
L $\frac{3}{5}$

Go on to the next page.

16 $10 - 2\frac{3}{7} = ?$

A $7\frac{4}{7}$ **B** $8\frac{3}{7}$

C $8\frac{4}{7}$ **D** $7\frac{3}{7}$

E None of these

17 $1\frac{7}{8} \div \frac{3}{4} = ?$

F $1\frac{10}{32}$ **G** $2\frac{1}{8}$

H $2\frac{1}{2}$ **J** $1\frac{1}{8}$

L None of these

18 $12 \div \frac{2}{3} = ?$

A 16
B None of these
C 2
D 8
E 24

19 $1.80 \div 1.5 = ?$

F 12
G 1.2
H .12
J .012
L None of these

20 $\frac{5}{8} \times \frac{32}{15} = ?$

A $\frac{37}{120}$ **B** $1\frac{6}{15}$

C $2\frac{2}{3}$

D None of these

E $1\frac{1}{3}$

21 $13\frac{2}{3} + 4\frac{7}{12} = ?$

F $18\frac{1}{4}$ **G** $17\frac{9}{15}$

H $17\frac{3}{4}$ **J** $18\frac{5}{12}$

L None of these

22 $7.2 \div .06 = ?$

A .012
B 12
C 1.2
D 120
E None of these

23 125% of 60 = ?

F 750
G 48
H 75
J 480
L None of these

24 40 is what per cent of 160?

A 40%
B 2.5%
C 4%
D None of these
E 25%

25 $\frac{1}{5}$% of 2000 = ?

F 40
G 4
H 400
J .04
L None of these

26 20 is 5 per cent of what number?

A 400
B 10
C 40
D 100
E None of these

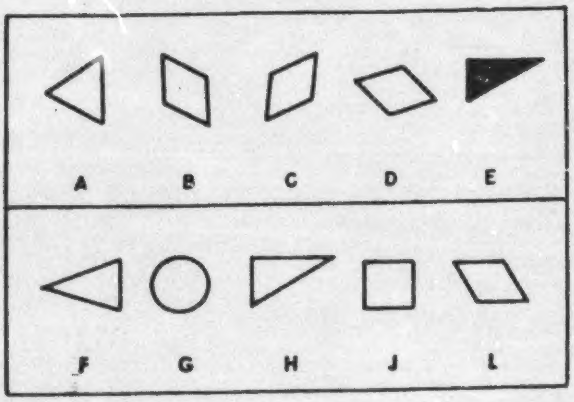
STOP. Wait here for further directions.

DIRECTIONS, PART V, CLASSIFICATION

In each group of drawings find the one which does not go with the others. Then look at the answer sheet, Part V, to see how the answer is marked.

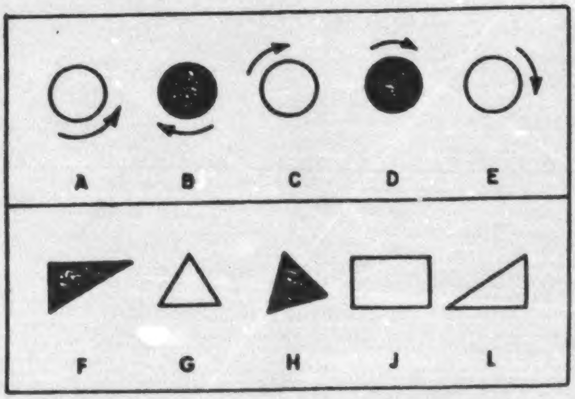
Y1

Y2



Y3

Y4



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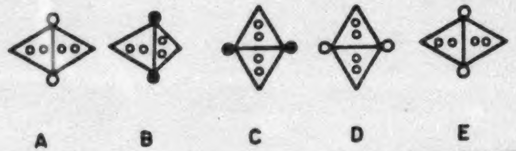
PART V, CLASSIFICATION

In each group of drawings find the one which does not go with the others.

1



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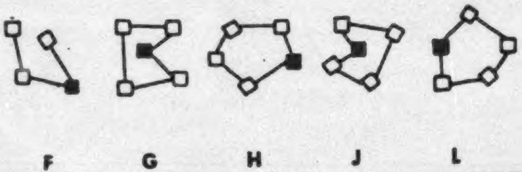

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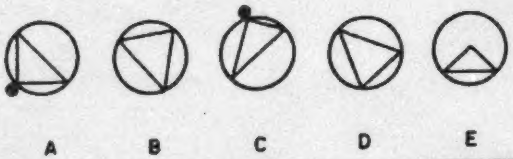

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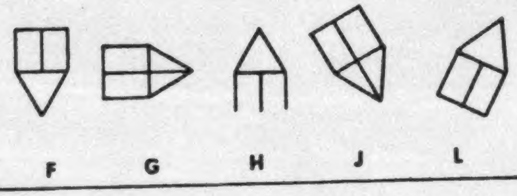

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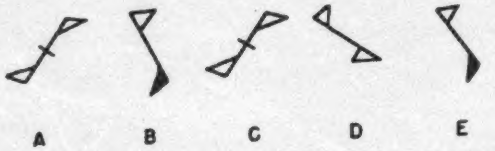

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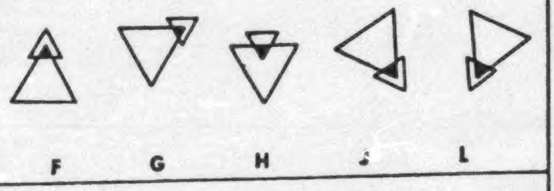

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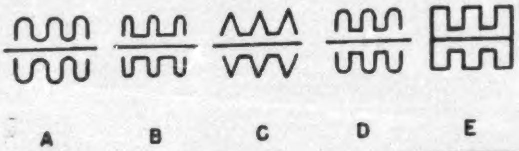

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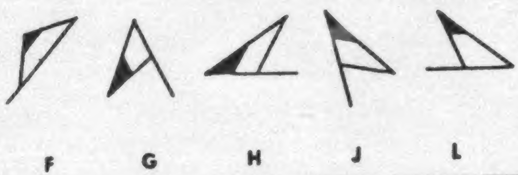

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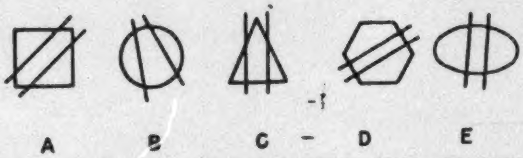

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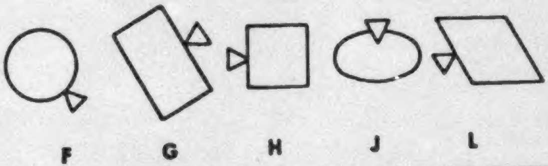

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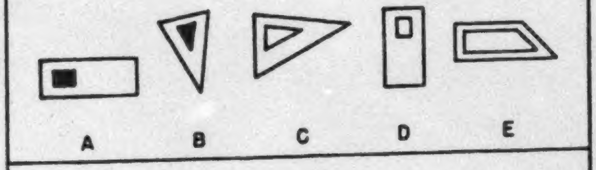

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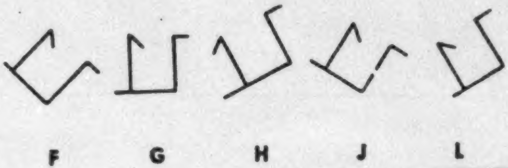

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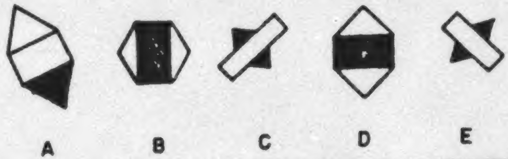

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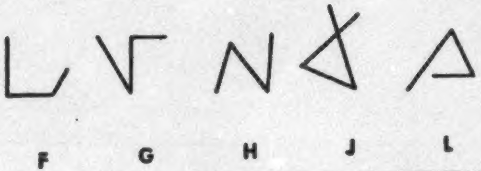

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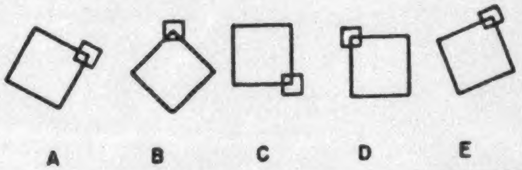

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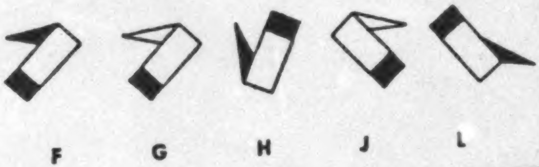

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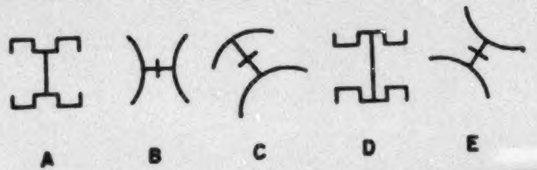

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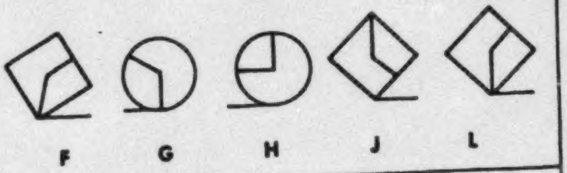

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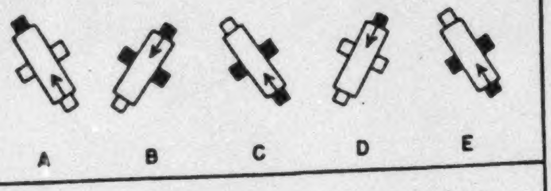

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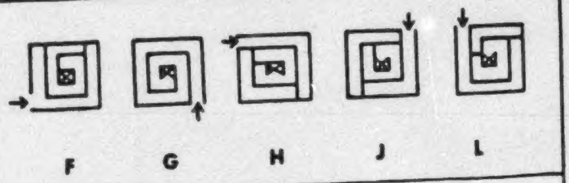

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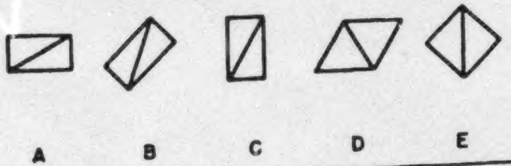

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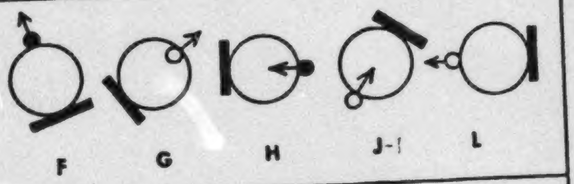

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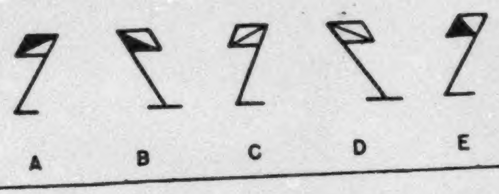

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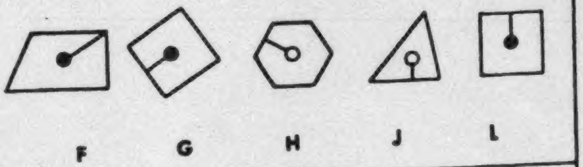

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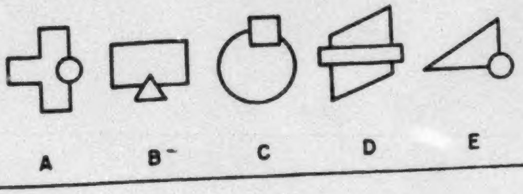

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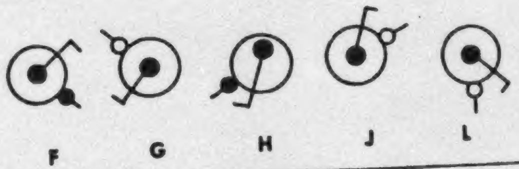

STOP. Wait here for further directions.

APPENDIX G

Sorting Test: Words/Concepts

SORTING TEST
Words/Concepts

RATIONAL NUMBER	有理數
X	
÷	
IDENTITY	恒等
ABSOLUTE VALUE	絶對值
∩	
+	
FACTOR	
DISTRIBUTIVE	分配
SLOPE	
LINE	線
SET	集
COMMUTATIVE	交換
–	
ELEMENT	元
PLANE	平面
RADICALS	根
EQUATION	方程
⩽	
EXPONENT	指數
GRAPH	圖表
√‾	
POINT	點
ASSOCIATIVE	結合
⩾	
POLYNOMIAL	多項式
COORDINATE	坐標
QUADRATIC	二次

APPENDIX H

Phase III Algebra Test Items

1. Factor: $4a^2 - 12ab + 9b^2$

2. $\sqrt{x^2} = ?$

3. Solve: $9x^2 = 3x + 2$

4. Find an equation of the line which contains the points (0,2) and (-1,3)

5. Simplify $\frac{2}{a-2} + \frac{1}{a^2-4} + \frac{3}{a+2}$

6. $x = \sqrt{x-2} + 4$, $x = ?$

7. $\frac{x-2}{x+3} \div \frac{x^2-4}{x+3}$

8. Graph the solution set $x^2 - 4 > 0$

9. $\frac{x^{2}y^{2}}{x^{2}y^{3}} = ?$

10. Graph $y \geq (x-2)^2 + 3$

11. Find two integers whose sum is 18 and whose product is 72.

12. An equation of the line which is parallel to the y-axis and which passes through the point (4, -3) is ______________?

13. Simplify $\sqrt{3}\,(2\sqrt{3} - \sqrt{2})$

14. Find the roots of $\frac{3}{2x-6} - \frac{2x-3}{2x^2-5x-3} + \frac{1}{2x+1} = 0$